Society of Production Engineering

ICPES 2025

40th INTERNATIONAL CONFERENCE ON PRODUCTION ENGINEERING - SERBIA 2025

DOI: 10.46793/ICPES25.466S

University of Nis Faculty of Mechanical Engineering

Nis, Serbia, 18 - 19th September 2025

REVIEW OF METHODS AND APPROACHES FOR EVALUATING MEASUREMENT UNCERTAINTY OF CMM

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Abstract: *For a long time, the evaluation of measurement uncertainty in coordinate measuring machines (CMMs) has been one of the most prominent research areas in the field of dimensional metrology. This is because coordinate measuring machines, whether equipped with contact or non-contact sensors, are dominant in assessing the conformity of workpieces with specifications. They are characterized primarily by a high degree of flexibility and accuracy, but they are also complex metrological systems with numerous sources contributing to measurement errors, i.e., measurement uncertainty. Numerous methods and approaches have been developed so far for the evaluation of CMM measurement uncertainty, and it is generally accepted that this is a highly challenging and complex task. Experimental methods, simulation-based methods, analytical approaches, as well as the ISO 15530 series of standards have proposed their own methodologies. The aim of this paper is to present the current state of research in this field. The authors of this paper have many years of experience in this area, and several case studies will be presented.*

Keywords: *dimensional metrology, CMM, quality control, accuracy, measurement uncertainty, accuracy.*

1. INTRODUCTION

Modern industrial demands increasingly call for the design of workpieces with extremely tight specifications in order to ensure the intended functionality of the final product. As a result and considering the inherent imperfections of manufacturing processes metrological systems used for quality control must operate as close to ideal conditions as possible. In other words, the quality of measurement results must be exceptionally high, allowing measurement imperfections to consume no more than 10 % of the given tolerance [1].

Due to their high flexibility and satisfactory accuracy, coordinate measuring machines (CMMs) have been the dominant measuring instruments for several decades when it comes to evaluating dimensional and geometrical characteristics of parts at the macroscopic level. The key performance parameter defined for these instruments is the maximum permissible error (MPE) when measuring geometrical tolerances typically in the form of a hemisphere to assess sensor error. This includes MPE_P for contact sensors in "point-by-point" measurement mode and $MPE_{Tij/\tau}$ for contact sensors operating in scanning mode, as well as MPE_E for reference length measurements [2]. For industrial-grade CMMs, these maximum permissible errors typically range from 1 to 5 µm. However, due to the complexity of these measuring systems, such parameters alone are insufficient to fully define the quality of the measurement results. Therefore, it is highly desirable to evaluate the measurement uncertainty for each specific measuring task.

A measurement result is considered complete only when accompanied by a statement of the uncertainty—i.e., an interval within which the true value is reasonably expected to lie. Given the complexity of both the measuring system and the measurement process itself, numerous contributing factors and their interactions must be considered, all of which can affect measurement uncertainty. The main challenge lies in identifying and quantifying all sources of standard uncertainty and incorporating them into an uncertainty model—if such a model can even be reliably formulated. So far, research into CMM measurement uncertainty has applied various approaches, including the Guide to the Expression of Uncertainty in Measurement (GUM), experimental methods, computational simulations, and expert judgment. Some of these approaches have been standardized through the ISO 15530 series of standards. The most recent research in this area is discussed in the referenced publication [3]. The following section provides an overview of key studies and findings in this field.

2. EVALUATION OF CMM MEASUREMENT UNCERTAINTY USING THE GUM APPROACH

The consensus among researchers and experts is that the GUM approach has significant limitations when applied to CMM measurements, primarily due to the number of assumptions that must be introduced. Alternative methods for evaluating measurement uncertainty often require extensive experimental procedures, access to calibrated reference workpieces, or a detailed virtual model of the measurement process and the behaviour of the CMM. These approaches are frequently costly and complex, especially when applied to CMMs operating in a production environment.

As a result, the GUM method remains the most practical option provided that its use yields a reliable estimate of uncertainty. In some cases, the Maximum Permissible Error (MPE) can be accepted as a measure of uncertainty, particularly when evaluating simple dimensional characteristics such as the distance between two points. Even in such cases, however, the GUM framework is used to assess the uncertainty of that measurement.

The main drawbacks of applying the GUM methodology to CMM uncertainty evaluation can be summarized as follows:

- Assumption of independence – introduced to avoid complex covariance terms;
- Standard deviation distribution – the conventional GUM method uses standard deviation as a measure of uncertainty;
- First-order Taylor approximation – the GUM approach relies on the first-order Taylor series expansion of the measurement function;
- Analytical relationship – the method assumes that the measurement process can be expressed as an analytical function of a set of input quantities, each associated with its own uncertainty;
- Systematic errors – the GUM framework also assumes that all known systematic effects have been identified and appropriately compensated for.

The lack of an analytical relationship between input quantities and the measurement result is one of the main reasons why the conventional GUM method is difficult to apply to CMM measurements. In the GUM-based approach, the starting point for determining measurement uncertainty in CMM applications involves formulating the equation of the substitute geometric primitive. However, commercial CMM software typically does not provide access to the equations of these reference (substitute) elements. To properly assess measurement uncertainty using GUM, it is necessary to develop independent software capable of generating both the reference equations and the extreme point coordinates. The application of GUM is most representative in the context of form error evaluation, since, as previously mentioned, the maximum permissible error can in some cases be considered as the measurement uncertainty—particularly when measuring lengths.

2.1 Evaluation of Measurement Uncertainty in Flatness Measurement on a CMM Using the GUM Method

The application of the GUM approach relies on principles of analytical geometry. The first step in evaluating the uncertainty of flatness measurement on a CMM is to define the reference plane from the sampled points. Since the relevant standard specifies that form error should be determined using the minimum zone method, the equation of the reference plane is obtained using a Bundle of Plains through One Point [4]. Flatness error is defined as the minimum distance between the points at the maximum (x_1, y_1, z_1) and minimum (x_2, y_2, z_2) deviation from the reference plane, as expressed in Equation (1):

$$\delta = \frac{(z_1 - z_2) - a(x_1 - x_2) - b(y_1 - y_2)}{\sqrt{1 + a^2 + b^2}}. \qquad (1)$$

To determine the uncertainty of the flatness error δ, it is necessary to evaluate the uncertainty and the propagation coefficient of each element in Equation (1). From this, it follows that the measurement uncertainty of the flatness error estimate is given by Equation (2):

$$\begin{aligned} u_\delta^2 = &\left(\frac{\partial\delta}{\partial x_1}u_{x1}\right)^2 + \left(\frac{\partial\delta}{\partial x_2}u_{y1}\right) + \left(\frac{\partial\delta}{\partial y_1}u_{z1}\right) + \\ &\left(\frac{\partial\delta}{\partial y_1}u_{x2}\right) + \left(\frac{\partial\delta}{\partial z_1}u_{y1}\right) + \left(\frac{\partial\delta}{\partial z_2}u_{z2}\right) + \\ &\left(\frac{\partial\delta}{\partial a}u_a\right) + \left(\frac{\partial\delta}{\partial b}u_b\right) + 2\frac{\partial\delta}{\partial a}\frac{\partial\delta}{\partial b}\rho_{ab}u_{ab}. \end{aligned} \qquad (2)$$

The uncertainty components denoted as u_{xi}, u_{yi} and u_{zi} represent the uncertainty of the sampled points, with the dominant influences arising primarily from the CMM's geometric errors and sampling system inaccuracies, as well as environmental conditions affecting both the CMM and the measurement process. The uncertainty components u_a, u_b, $u_a u_b$ and ρ_{ab} correspond to the uncertainty associated with the algorithm used to derive the substitute geometry. The coefficients defining the orientation of the reference plane, a and b, are correlated and represent the correlation coefficient. For a detailed case study on the evaluation of measurement uncertainty in flatness measurement on a CMM using the GUM method, refer to the study [5]. The same methodology can be applied to assess measurement uncertainty in other form errors on CMMs, such as roundness, straightness, and cylindricity.

3. EVALUATION OF CMM MEASUREMENT UNCERTAINTY USING ISO 15530-3:2011 STANDARD

The evaluation of CMM measurement uncertainty according to the guidelines of ISO 15530-3:2011 [6], is considered the most reliable method and is often used as the reference for validating simulation-based approaches [7]. This method involves repeated measurement—at least 20 times—of a calibrated workpiece that is geometrically identical to the part being inspected, positioned at different locations and orientations within the CMM's measurement volume. Furthermore, the conditions under which the calibrated part is measured must be identical to those used for the actual workpiece, including the same probe configuration, measurement strategy, and environmental conditions. The differences between the measured results and the known calibrated values of the reference part are used to estimate the measurement uncertainty. The uncertainty components arise from the measurement procedure itself, the calibration of the reference part, and variations in the measured features such as deviations in form error, thermal expansion coefficients, and surface texture. The expanded measurement uncertainty is calculated according to Equation (3):

$$U = k\sqrt{u_{cal}^2 + u_p^2 + u_b^2 + u_w^2} \quad (3)$$

where k is the coverage factor used to calculate the expanded uncertainty; u_{cal} represents the standard uncertainty associated with the calibration of the reference part, as stated in the calibration certificate, u_p is the standard uncertainty derived from the experimental measurements and includes uncertainty components related to the CMM hardware, u_b represents the standard uncertainty associated with systematic errors in the measurement process and u_w is the standard uncertainty linked to material and manufacturing variations. Since the systematic error b, defined as the difference between the mean value obtained from the CMM measurements and the calibrated value of the observed feature typically remains uncompensated, the total measurement result is expressed using Equation (4):

$$Y = y - b \pm U \quad (4)$$

where Y represents the total measurement result, y is the result obtained from the CMM, b is the systematic error and U is the expanded measurement uncertainty. An application of this approach to a specific case can be found in [8]. However, the standard in question has several limitations. First, the standard uncertainty u_w consists of two components, where u_{wp} accounts for uncertainty arising from the manufacturing process [9]. This component is particularly difficult to quantify, as it is inherent to the machining process of the workpiece and is therefore often neglected. Second, a point of ongoing discussion in the scientific community concerns the value of L which appears in the equations for the standard uncertainties u_w and u_b in the case of form error measurements [10]. Specifically, when measuring deviations from ideal form, the value of L is considered to be zero. As a result, both standard uncertainties are also evaluated as zero, which can misrepresent the true uncertainty. Third, the standard requires the application of the same measurement strategy across multiple orientations and positions of the workpiece within the CMM's working volume. This approach accounts only for uncertainties arising from the CMM hardware, such as geometric errors and sensor inaccuracies. To address these limitations, the International Organization for Standardization has worked on a draft of ISO 15530-2, which proposes multiple measurement strategies with variations in part orientation and position within the CMM's measurement space. Additionally, the draft standard suggested the use of uncalibrated parts, which may be one of the main reasons it was never formally published. Nevertheless, ISO 15530-3:2011 can be extended by incorporating various measurement strategies, using probing styli of different diameters, and varying scanning speeds when operating in scanning mode. In such cases, the application of design of experiments (DoE) techniques proves highly beneficial.

4. APPLICATION OF SIMULATION METHODS FOR EVALUATING CMM MEASUREMENT UNCERTAINTY

Numerous models have been developed for evaluating measurement uncertainty on CMMs, with most relying on simulation-based principles. The core idea behind these methods is to model both the machine and the measurement process in order to replicate the behaviour of a real coordinate measuring machine. Two main approaches can be distinguished: the first, more straightforward approach involves using simulation to propagate the uncertainty of sampled points through a model that represents the equation of the substitute geometric primitive [11]. The second, more complex approach is based on constructing a virtual coordinate measuring machine (VCMM), which is a software-based solution designed to digitally replicate the operation of a specific CMM [12]. The use of simulation methods has been standardized under ISO 15530-4:2008, and several commercial software tools—known as Uncertainty Evaluating Software (UES)—are available for this purpose. These tools primarily use Monte Carlo simulation as the computational engine. Currently, there are only a few commercial and non-commercial software solutions available for uncertainty evaluation, most of which are developed and maintained by leading national metrology institutes and research centers. UES tools are capable of evaluating valid uncertainties for a wide range of measurement tasks, and as such, both the measurement procedures and the simulations can be considered as part of traceable calibration workflows [13]. Below are some representative examples of simulation-based methods for evaluating CMM measurement uncertainty:

- Virtual Coordinate Measuring Machine (VCMM) – The first software solution developed for evaluating the measurement uncertainty of coordinate measuring machines was named the Virtual Coordinate Measuring Machine [3]. This software tool was developed in the 1990s by PTB (Physikalisch-Technische Bundesanstalt, Germany)
- Virtual Coordinate Measuring Machine MCM PK – A virtual coordinate measuring machine named MCM PK was developed at the Cracow University of Technology (Poland) [12]. The operating principle of this software solution is similar to that of the classical VCMM, although different techniques were applied to model the real behaviour of the CMM;
- Virtual Coordinate Measuring Machine Extended with Form Error Uncertainty Source – At the Catholic University of Leuven (Belgium), a software solution was developed that, in addition to accounting for hardware-related CMM errors, also considered form error as one of the more significant sources of measurement uncertainty in CMM measurements [14]. It was also found that existing commercial software solutions at the time were not capable of incorporating this factor into uncertainty evaluation;
- Expert CMM (ECMM) – A group of authors [15] introduced a concept for evaluating CMM measurement uncertainty using Monte Carlo simulation, referred to as the Expert Coordinate Measuring Machine (ECMM). The operational principle of ECMM consists of two conceptual steps: first, the estimation of uncertainty for the individual point coordinates; and second, the propagation of this uncertainty through the part measurement program;
- Simulation by Constraints – PUNDIT/CMM – The National Institute of Standards and Technology (NIST) in the United States developed a simulation-based method for evaluating CMM measurement uncertainty, known as Simulation by Constraints (SBC) [16]. This approach allows both the CMM and its sensor to be modeled using results from tests prescribed by ISO 10360.

4.1 Application of Monte Carlo Simulation for Evaluating Measurement Uncertainty in Flatness Measurement on a CMM

This simulation approach is based on a simplified application of the Monte Carlo method, without the use of a virtual model of the coordinate measuring machine (CMM). In this model, the uncertainty of a sampled point is not described through calibration procedures, but rather through the corresponding repeatability of the sampled point, determined experimentally. The model for evaluating measurement uncertainty in flatness measurements is illustrated in Figure 1. The inputs to the simulation model are probability distribution functions of the coordinates of the sampled points, obtained from ten repeated measurements on an actual, randomly selected workpiece. The repeatability of each sampled point (*x*, *y* and z coordinates) is described using a normal distribution, where the mean values $\overline{x_i}$, $\overline{y_i}$, $\overline{z_i}$ and standard deviations $\overline{\sigma x_i}$, $\overline{\sigma y_i}$, $\overline{\sigma z_i}$ are calculated from the experimental sampling data. Such input probability distributions of the point coordinates inherently include uncertainties arising from the interaction of the CMM hardware with the environment. All geometric and sensor-related errors, whether random or systematic, are embedded in the defined probability distribution function that describes the sampled point. Although defining the input based on the repeatability of the sampled point may pose challenges due to the need for repeated measurements, the time required for acquiring this repeatability is comparable to that needed for modeling geometric and sensor errors through measurements of specific calibration workpieces. The accuracy of such inputs to the simulation model is inherently higher, as they are based on real-world measurement conditions. Furthermore, the simulation model incorporates various sampling strategies, i.e., different numbers of sampling points distributed according to the Hammersley distribution on the measured surface. Specifically, this model for determining the flatness of a workpiece included sets of b=10, 20, 40, 75 and 140. It is assumed that flatness error increases with the number of sampling points in the measurement strategy, while the measurement uncertainty decreases. In order to investigate how flatness error and measurement uncertainty behave depending on the position of the workpiece within the CMM measuring volume, the *x*, *y*, and *z* coordinates were sampled according to a predefined sampling plan at five different positions within the measuring volume. Through these experiments, the CMM measuring space was calibrated for flatness measurements, providing the user with insights into specific regions of the machine where higher or lower flatness errors and, correspondingly, lower or higher measurement uncertainties may occur. Once the input parameters were defined, the Monte Carlo simulation was carried out. Due to its stochastic nature, the calculated uncertainty (expressed as a confidence interval) will vary between simulation runs. After generating M virtual samples from the defined input distribution functions, a criterion must be applied to determine the substitute geometry, i.e., to estimate the reference planes and flatness errors. For this purpose, a novel methodology based on the Minimum Zone (MZ) method—Bundle of Plains through One Point was employed. Specifically, the first sampled values (x_1, y_1, z_1)...(x_b, y_b, z_b) are taken to define coordinates for *b* points. Through these b points, using the MZ method with rotation through a single point, the equation of the plane and the flatness error are determined. Then, the second set of sampled values (x_1, y_1, z_1)...(x_b, y_b, z_b) is used to generate a new set of b points, which are input into the algorithm to estimate the reference plane and flatness error again. This procedure is repeated for all M equations of planes, resulting in M flatness error values. Additionally, the process is repeated for several positions within the available CMM measuring volume. Based on the M obtained flatness error values, a frequency distribution is constructed in the form of a histogram, from which the 95 % confidence interval is used to determine the expanded measurement uncertainty. After the simulation is completed, focus is placed on the influence of different numbers of sampling points and the position of the workpiece within the measuring volume on the values of expanded uncertainty and flatness error. Further results of this investigation are presented in [17]

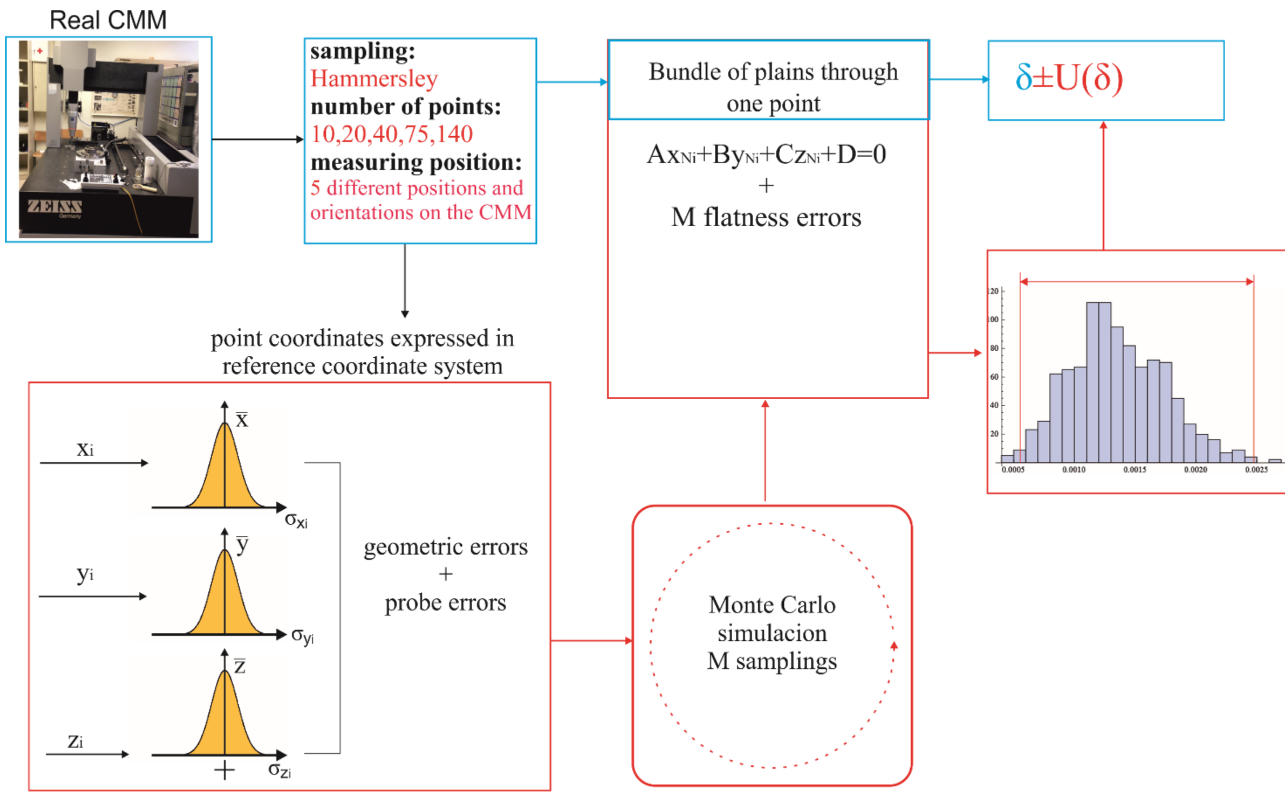

Figure 1. Model for estimating measurement uncertainty of CMM during flatness measurements based on Monte Carlo simulation

5. CONCLUSION

The accurate evaluation of measurement uncertainty in coordinate measuring machines (CMMs) is a critical factor in ensuring the reliability and traceability of dimensional measurements in modern manufacturing. This paper has presented a comprehensive review of existing methods and approaches used for uncertainty assessment, including ISO-based approaches, statistical methods, Monte Carlo simulations, and numerical modeling techniques.

Each method carries specific advantages and limitations depending on the measurement context, the complexity of the part, and the measurement environment. While ISO 15530 series offers a standardized framework—especially through substitution and simulation methods—alternative approaches such as Monte Carlo methods provide greater flexibility in complex and non-standard applications.

It is evident that the integration of multiple methods, combined with advanced software and sensor technologies, represents a promising direction for future development. Furthermore, the adoption of digital twins and machine learning techniques holds significant potential for real-time uncertainty estimation and process optimization.

ACKNOWLEDGEMENT

This research has been supported by the Ministry of Science, Technological Development and Innovation (Contract No. 451-03-137/2025-03/200156) and the Faculty of Technical Sciences, University of Novi Sad through project “Scientific and Artistic Research Work of Researchers in Teaching and Associate Positions at the Faculty of Technical Sciences, University of Novi Sad 2025” (No. 01-50/295).

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