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VIRTUAL REALITY ENVIRONMENT FOR TEACHING ASSEMBLY SEQUENCE ANALYSIS TO PRODUCTION ENGINEERING STUDENTS

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Abstract: *Assembly sequence analysis represents the first step in assembly process design and plays a very important role in achieving optimal production time and cost, which is of great importance in large-scale and mass production. There are several different approaches to address this issue. As a rule, they start from the analysis of product components and their mutual liaisons (contacts, i.e., mates or joints), followed by generation of all feasible assembly sequences and, finally, selection of the optimal sequence according to appropriate criteria. However, mastering these tasks can be challenging for students who encounter them for the first time, and particularly when physical interaction with the product is limited. To facilitate and enhance students' understanding of these methods by making the learning process more intuitive and easier, while also assisting educators in effective teaching, the Virtual Reality (VR) technology can be used. This paper presents a VR-based learning workflow for teaching methods for assembly sequence analysis to production engineering students. The workflow contains predefined tasks that students follow in VR to achieve the intended learning outcomes without the need for significant manual effort. For this workflow several VR environments are developed. In addition to these environments, the paper presents the analysis of the students' experiences in their utilization.*

Keywords: *Assembly sequence analysis, Cut-set method, Bourjault method, VR technology, Production engineers' education*

1. INTRODUCTION

The assembly process represents the final and one of the most critical stages in manufacturing, during which individual components are joined to form a functional product. In addition to its direct influence on product performance and quality, assembly has a major impact on the overall efficiency and cost-effectiveness of production. In traditional industrial manufacturing, assembly-related activities account for approximately 20–30% of total production costs and between 40–60% of total production time [1]. These figures highlight the importance of assembly process design, which becomes especially critical in large-scale and mass production.

The first step in assembly process design is assembly sequence analysis, which aims to determine the order in which parts should be assembled to form the final product. This analysis has three different approaches: finding all feasible sequences, finding all linear feasible sequences, and finding one feasible or one linear feasible sequence [2]. The first approach is the most complex, but it offers the greatest potential for the assembly process optimization. One of the first algorithms for generating all feasible assembly sequences was proposed by Bourjault [3]. Another notable approach in this field is the Cut-set method, introduced by Homem de Mello and Sanderson [4]. Despite being introduced many years ago, both methods are still actively used and studied, with ongoing research focused on enhancing their performance and integrating them into automated software tools. The Cut-set method has recently been applied in various contexts, including the generation of optimal disassembly sequences for End-of-Life products [5], the decomposition of complex assembly tasks in collaborative environments [6], and the identification of subassemblies within assembly planning [7]. Additionally, the core principles of the Bourjault method have inspired structured, question-based approaches aimed at analyzing disassembly paths and precedence relationships between components [8].

Considering their significance in defining assembly process steps and optimizing production performance, it is of great importance that future mechanical/production engineers acquire a deep understanding of these methods. Consequently, Bourjault and Cut-set methods are integrated into the curriculum of the Assembly Technology course at the Faculty of Mechanical Engineering, University of Belgrade. Due to their complexity, teaching these methods to production engineering students using traditional approaches brings about significant challenges primarily related to students' understanding of their principles. As a potential solution, we have developed a Virtual Reality (VR) based workflow to make these methods, as well as the whole assembly sequence analysis more accessible to students. The VR-based workflow and the initial results of its implementation are presented in this paper.

The remainder of the paper is structured as follows. In Section 2 we briefly introduce the methods used for assembly sequence analysis including Cut-set and Bourjault methods. Section 3 is related to the learning objectives and intended learning outcomes of the proposed workflow, whereas the workflow itself is presented in Section 4. Section 5 refers to students' experiences regarding the workflow utilization. Finally, Section 6 presents the conclusions of the paper.

2. ASSEMBLY SEQUENCE ANALYSIS METHODS DESCRIPTION

The starting point in assembly sequence analysis is the generation of Liaison Diagram. This diagram provides information about components of the product and shows the relations between them in a form of graph. In this graph, nodes represent the components of the product, while the liaisons (contacts, i.e., mates or joints) are shown as links labeled with numbers [2]. A Liaison Diagram for a simple exemplary product is shown in Fig. 1.

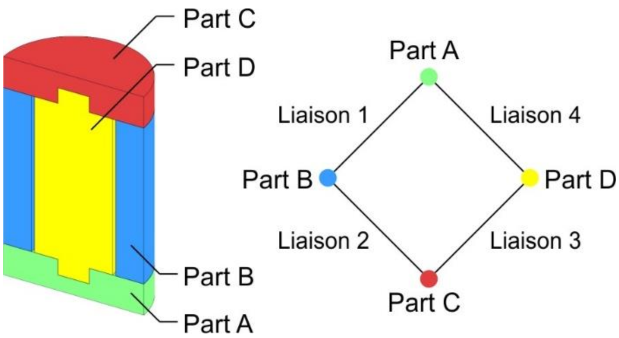

Figure 1. Simple product Liaison Diagram

There are certain assumptions on which the construction of the Liaison Diagram is based [9]:

1. Parts are rigid,
2. The liaison can be formed once and only once, and the relative position and orientation of the parts in the

(sub-)assembly cannot be changed once the assembly operation is finished.

The Liaison Diagram represents a starting point for the generation of all feasible sequences of product assembly. The following step is either Cut-set method or Bourjault method using which precedence questions are asked and answered in predefined manner to get the precedence relations which define the order in which parts must be assembled. Both methods (Cut-set and Bourjault) have the same objective and similar outputs and can be used according to the designer's preferences.

2.1 Cut-set method

Cut-set method assumes that disassembly sequences are the inverse of the possible assembly sequences. Therefore, the method evaluates whether each possible subassembly can be disassembled from the main assembly or another subassembly. During the evaluation the following precedence question is asked [9]:

From an assembled state, can subassembly S_j be disassembled from subassembly S_i?

It is important to note that a subassembly can consist of a single part [9]. There are three possible answers:

1. **Not applicable (N/A)** – the parts within set S_i or S_j do not form a connected subgraph, i.e., they are not physically connected.
2. **Yes** – The subassembly S_j can be physically disassembled from subassembly S_i. For example, based on Fig. 1, the subassembly S_j=(Part C, Part D) can be disassembled from the subassembly S_i=(Part A, Part B).
3. **No** – The subassembly S_j cannot be physically disassembled from subassembly S_i. For example, from Fig. 1, the subassembly S_j=(Part D) cannot be disassembled from the subassembly S_i=(Part A, Part B, Part C).

If the answer to the precedence question is **No**, the subassembly set S_i is recursively broken down by reassigning each of its components to a new subassembly S_j, while the remaining components form the new S_i. The precedence question is then repeated for each of these new combinations.

2.2 Bourjault method

Bourjault method determines the possibility of establishing each liaison at a given stage of assembly. The precedence question posed by this method is:

Can liaison i be established if liaisons from set B have previously been established?

This question is referred to as R-question and can be written in the form R(*i*; *B*). Similar to the Cut-set method, there are three possible answers to the question above [9]:

1. **Not applicable (N/A)** – The liaison *i* is automatically established by establishing liaisons from set *B*. For example, the answer to the question R(1; 2, 3, 4) for the assembly in Fig. 1 is **N/A** because liaison *i*=1 is automatically established by establishing liaisons from set *B*=(2, 3, 4).
2. **Yes** – The liaison *i* can be established even though the liaisons from set *B* have already been established. For example, for the assembly in Fig. 1, the answer to the question R(1; 2, 3) is **Yes** because the liaison *i*=1 can be established even though the liaisons from set *B*=(2, 3) have already been established.
3. **No** – The liaison *i* cannot be established if the liaisons from set *B* have already been established. For example, for the assembly in Fig. 1, the answer to the question R(1; 3, 4) is **No** because the liaison *i*=1 cannot be established if the liaisons from set *B*=(3, 4) have already been established.

If the answer to the R-question is N/A or No, the procedure continues by removing the liaisons from set *B* one by one and repeating the question. On the other hand, if the answer is Yes the procedure is stopped for the given liaison set *B*.

2.3 Precedence relations

The outputs of both methods (Cut-set and Bourjault), obtained by addressing the precedence question, are precedence relations. These relations define the order in which subassemblies should be assembled. They are derived from precedence questions for which the answer is **No** and can be expressed in the following form:

$$R(A;B) = NO \rightarrow A \geq B \quad (1)$$

where $A \geq B$ indicates that all liaisons from set A must be made before liaisons from set B [2].

2.4 Liaison Sequence Diagram

The Liaison Sequence Diagram provides a graphical representation of all feasible assembly sequences based on the precedence relations obtained from the Bourjault or Cut-set method. It represents a state diagram, where assembly states are depicted as tables and transitions between these states are illustrated with connecting lines. The number of cells in each table corresponds to the number of liaisons within the assembly. Empty cells indicate liaisons that have not yet been established, whereas filled denote liaisons that have already been completed [9]. Paths through this diagram, staring from the state in which all fields are empty (no liaison made) to the final state in which all fields are filled (all liaisons made) represent different assembly sequences. An example of Liaison Sequence Diagram is shown in Fig. 2.

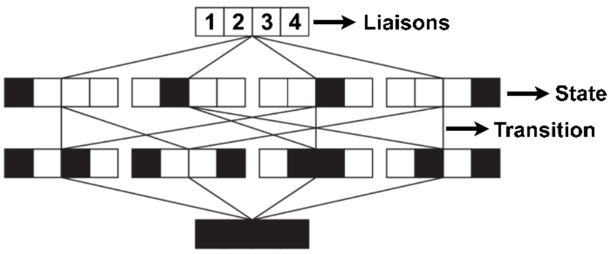

Figure 2. Simple product Liaison Sequence Diagram

When all feasible assembly sequences are generated and represented using the Liaison Sequence Diagram, following the predefined rules primarily related to Design-for-Assembly [10], the designer selects the most suitable assembly sequence to be implemented in assembly process. Learning workflow that will be presented in this paper intends to facilitate students' mastering and understanding of the described methods.

3. LEARNING OBJECTIVES AND OUTCOMES

The objectives of the VR-based learning workflow presented in this paper are to support the development of different types of knowledge, including factual knowledge – **F** (different approaches to assembly sequence design, key terminology, definitions and rules related to the Liaison Diagram, the Cut-set and Bourjault method, and the Liaison Sequence Diagram);

Table 1. ILOs with associated knowledge type (KT)

KT	ILO description
F	Capability to choose suitable technique for assembly sequence design – I1
F, P	Understanding the liaisons (mates and joints) between parts in the product – I2
F, P	Understanding physical constraints and relations between parts and subassemblies within product – I3
C	Understanding the influence of product design on assembly process – I4
C, P	Capability to define precedence of assembly operations within the process – I5
F, P	Capability to define all feasible assembly sequences for the selected product – I6
F, C	Capability to choose the most suitable assembly sequence – I7
P, M	Capability to apply VR tools to interact with virtual products and to extract useful information from a product by exploring it in virtual environment – I8
M	Capability to self-evaluate their own learning achievements – I9

conceptual knowledge – **C** (understanding the relations among key elements in the assembly sequence design, as well as the influence of the assembly sequence on the overall assembly process); procedural knowledge – **P** (understanding how to carry out certain tasks and procedures of assembly sequence design and to implement the corresponding methods); metacognitive knowledge – **M** (applying VR tools for solving assembly sequence design problem, as well as evaluating one's own understanding of the methods and validating the results through virtual experimentation).

Each type of outlined knowledge helps students achieve specific Intended Learning Outcomes (ILOs) that are listed in Table 1. These ILOs define the expected results of the learning process in terms of students' performance.

4. WORKFLOW DESCRIPTION

The developed workflow for teaching and learning the assembly sequence analysis in VR environment consists of eight learning activities – tasks (T), as shown in Fig. 3.

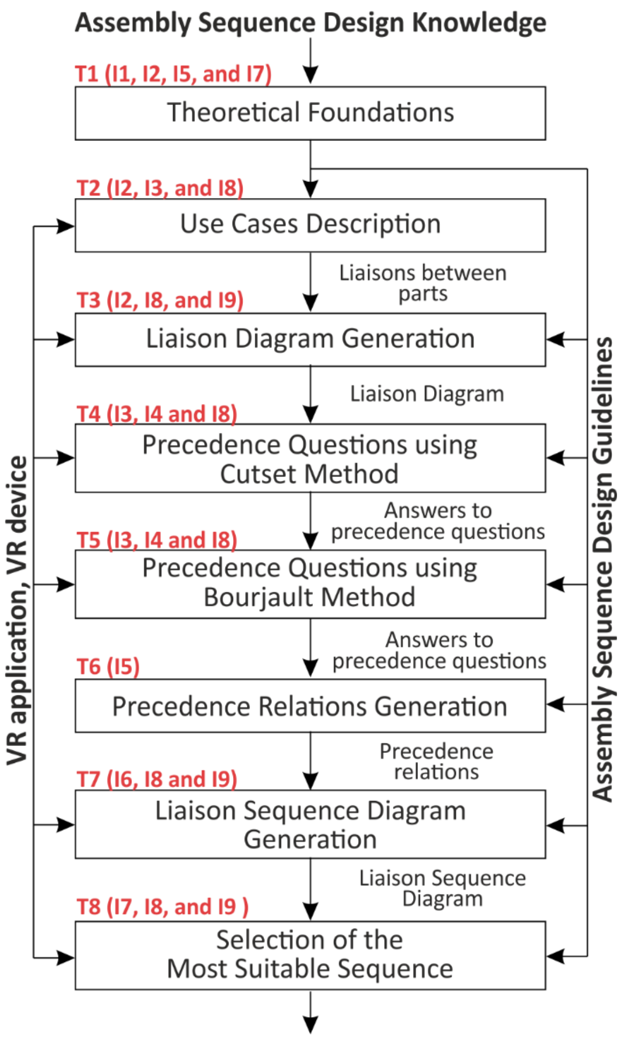

Figure 3. Learning activities within workflow

These tasks are designed to help students gain the intended knowledge and to achieve the ILOs – the ILOs achieved using certain task are presented in Fig. 3 in parentheses.

The development involved Unity3D v6000.0.3f1 as the main platform, SolidWorks for 3D modeling, and Oculus Rift S for immersive interaction. To prevent the collision between parts and to provide students with close to real-world experience and interaction between parts during assembling/ disassembling in VR, all parts within the model contain colliders. Since most of the parts in the selected use case are concave and some of them have complex form, creation of colliders was not a straightforward task. Unity's standard tools were not suitable for collider generation, and the open-source CoACD.unity library was used to create accurate mesh colliders and enable realistic physical interaction in VR. The details regarding this library can be found in [11].

For the workflow, we selected a step motor as the use case, providing a practical example of how the methods can be implemented in a realistic assembly scenario. The step motor is reduced to six parts/subassemblies labeled A–F, as shown in Fig. 4.

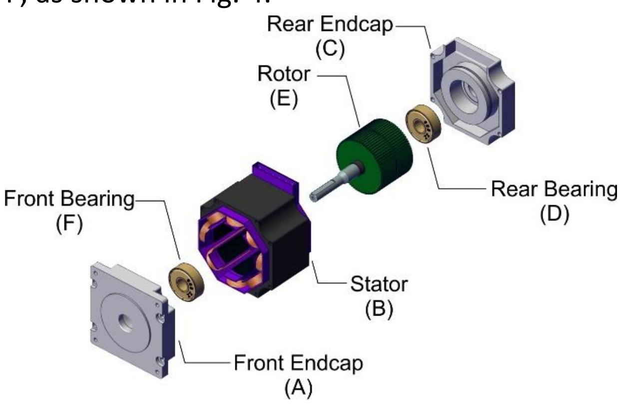

Figure 4. Exploded view of the step motor

Within T1 students are, in a traditional teaching manner, introduced to the fundamentals of assembly sequence design, including the advantages and shortcomings of identifying all feasible assembly sequences for designing assembly systems. This task focuses on mastering the theoretical concepts outlined in Section 2 and achieving I1, while simultaneously contributing to I2, I5, and I7.

In T2 students interact with a step motor assembly in a VR environment (Fig. 5). Through this process, they explore the structure of the product and gain better insight into the physical relations between its parts. Also, students are provided with a technical drawing and a 3D exploded view of the step motor. Within this task, students obtain elements of I2, I3, and I8.

Based on the VR experience from T2, the technical drawing, the 3D exploded view, and cutaway VR model of the step motor that can be assembled and disassembled, students are expected to generate a Liaison Diagram. In VR environment, they construct the diagram by placing links on a virtual canvas, as shown in Fig. 6. After completing their Liaison Diagram, students have the option in VR to view the correct version of Liaison Diagram and compare it with their own. If students notice any discrepancies between their Liaison Diagram and the correct one, they can further explore the product to identify the cause of their mistake. This task contributes to ILOs I8 and I9, whereas I2 is fully achieved within T3.

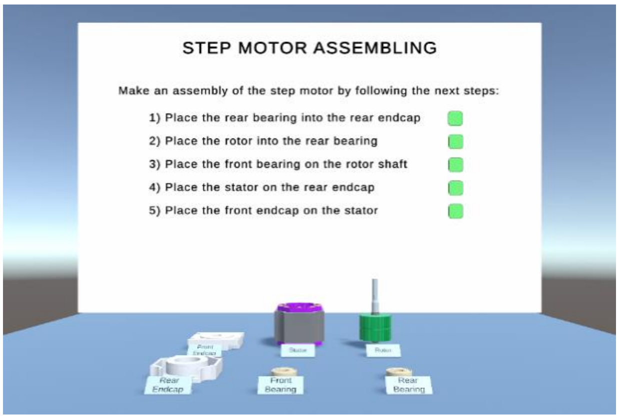

Figure 5. Interaction with the product in VR

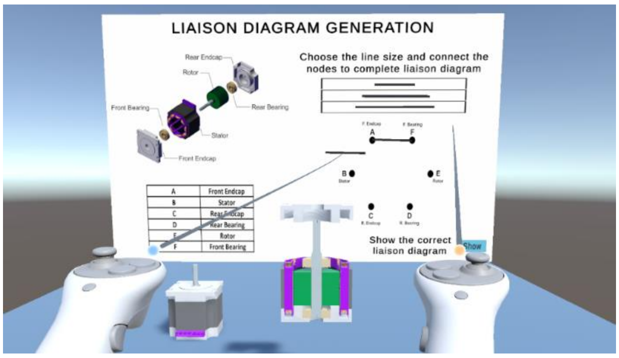

Figure 6. Liaison Diagram generation

In T4 students use the VR application to ask and answer precedence questions by applying the Cut-set method. The VR scene created for this task is shown in Fig. 7. The procedure for asking and answering precedence questions begins with defining the subassemblies S_i and S_j, which are generated using checkboxes on the central canvas (Fig. 8). Simultaneously, the nodes in the Liaison Diagram and the parts in the virtual product model that belong to subassembly S_j are automatically highlighted in red to visually distinguish between subassemblies S_i and S_j (Fig. 9).

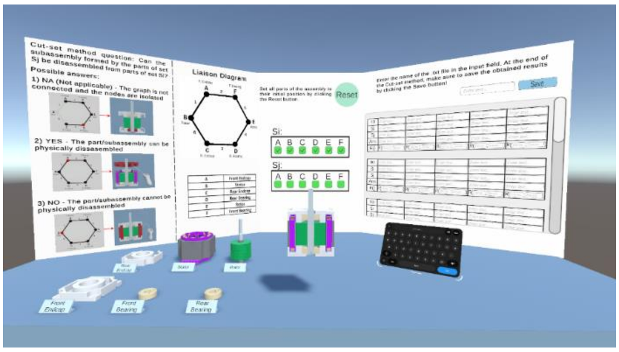

Figure 7. VR scene for Cut-set method

Figure 8. Checkboxes for generating subassemblies

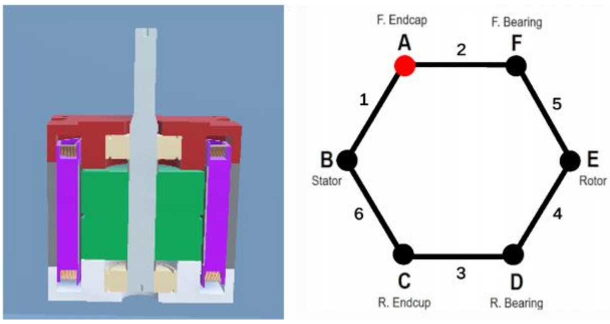

Figure 9. Automated highlighting of part and node

In the next step, students attempt to disassemble subassembly S_j from subassembly S_i on the virtual model of the product, to determine the answer to the precedence question defined by the Cut-set method (Fig. 10). The answers to the questions are entered into the table on the right canvas (Fig. 7) using a virtual keyboard and then stored in a .txt file. T4 contributes to I3, I4, and I8.

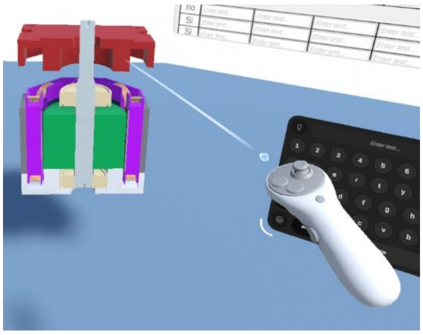

Figure 10. Disassembly trial

Task T5 refers to Bourjault method, and also contributes to I3, I4 and I8. In this task, students again use the VR application to ask and answer precedence questions, but this time by applying the Bourjault method. The VR scene related to this task is shown in Fig. 11.

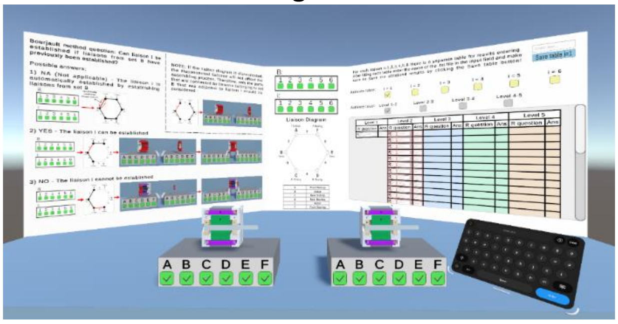

Figure 11. VR scene for Bourjault method

The procedure for asking and answering precedence questions begins with defining set *B* and liaison *i* from the R-question using checkboxes on the central canvas. This selection automatically generates corresponding Liaison Diagram in which liaisons that are not within set *B* are removed, and the liaison *i* is marked red; this is also displayed on the same canvas (Fig. 12).

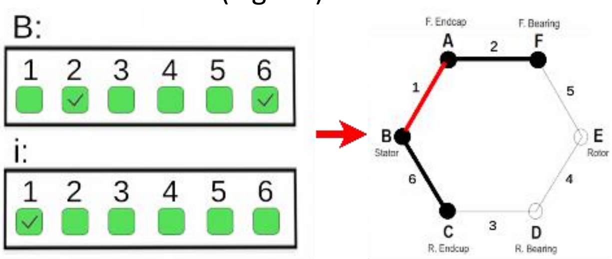

Figure 12. Checkboxes for defining set *B* and liaison *i* and corresponding Liaison Diagram

In front of the canvas, there are two instances of the step motor assembly. Each part of these assemblies can be activated or deactivated using the checkboxes located below them. Based on automatically generated Liaison Diagram, students activate/deactivate relevant parts and try to assemble these subassemblies (Fig. 13). Through this process, they derive the answer to the corresponding R-question. The parts between which the liaison *i* is established are highlighted in red.

Similarly to the Cut-set method, students enter their answers into a table on the right canvas (Fig. 11) using a virtual keyboard. Afterwards, these answers are saved in a .txt file for later assessment.

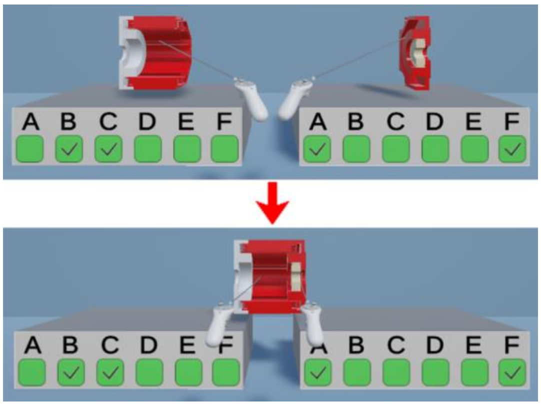

Figure 13. Assembly trial

To generate precedence relations in T6 students analyze the answers to the precedence questions obtained in tasks T4 and T5. These relations are generated separately for each method (Cut-set and Bourjault) and subsequently compared. In case of any differences between two sets of precedence relations, students should return to tasks T4 and T5 and try to discover their cause. During this task, students achieve I5.

Based on the precedence relations generated in task T6, within T7 students construct the Liaison Sequence Diagram for the step motor. After that, they immerse themselves in the VR environment to compare their diagram with a predefined reference version (Fig. 14). If they find differences between the diagrams, students should further explore the product, review its parts and connections, and try to understand where the error came from. In T7 students achieve I6, and this task contributes to I8 and I9.

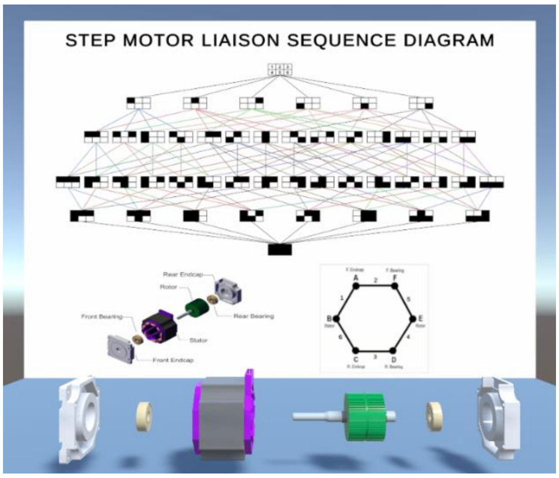

Figure 14. VR scene for Liaison Sequence Diagram analysis

As a final step, in T8 students test various selected assembly sequences in the VR environment shown in Fig. 14, and based on specific criteria (e.g., stability of subassemblies, the need for complex equipment, potential part damage) along with their own engineering reasoning, they select the most suitable assembly sequence. This is the final task in which I7, I8, and I9 are achieved.

5. STUDENTS EXPERIENCE

Despite the clearly defined rules of the methods used for assembly sequence analysis, and particularly for Cut-set and Bourjault methods, understanding their practical application can be challenging for students when a traditional teaching approach is used. This challenge becomes even greater if students do not have access to a physical model of the product, making the assembly sequence design more abstract and less intuitive. In some cases, even though a physical model of a product is available, certain factors including tight fits, size and weight of the product parts can complicate the product analysis and assembly/disassembly process. To address these challenges, the presented VR workflow offers meaningful support for learning and teaching the complex process of generating all feasible assembly sequences for a given product. The workflow enables students to better understand the subject matter and contributes to the learning outcomes through several interactive elements, such as:

1. Interaction with the product,
2. Easier engagement and improved understanding of the considered methods,
3. Easier Self-Evaluation.

Using the presented VR environment students interact with a 3D model of the step motor, freely assembling and disassembling it. The use of VR helps overcome some of the practical difficulties associated with real-world assembly that hinder students' concentration on mastering the assembly sequence design analysis techniques and direct their attention to alternative problems. These difficulties are related to applying relatively high forces due to tight fits (e.g., when mounting bearing) and the influence of the rotor's magnetic field when assembling other ferromagnetic parts. The seamless interaction with the product parts is very important for all steps in the process including generating the Liaison Diagram, asking and answering precedence questions, analysis of all feasible assembly sequences and selecting the most suitable one.

Specially designed VR scenes include features that simplify the learning process leading to easier engagement and improved understanding. These include the automatic generation of the Liaison Diagram using checkboxes for selecting either parts (Cut-set method) or liaisons (Bourjault method), red highlighting of parts creating liaison (Bourjault method) or the parts to be disassembled (Cut-set method), and structured tables for entering and reviewing precedence questions and answers for both methods.

The development of critical and self-critical thinking is one of the key objectives in engineering education. The presented workflow supports this process through student self-evaluation, particularly during the generation of the Liaison Diagram and the Liaison Sequence Diagram.

Students of the course in Assembly Technology at the University of Belgrade – Faculty of Mechanical Engineering had the opportunity to use the presented learning workflow within co-design lab in Erasmus+ XREN project ("Extended reality tools to support learning activities in engineering"). Six students participated in the co-design lab.

Table 2. Excerpt of the results of students' survey (Avg – average grade on Likert scale, STD – Standard Deviation on Likert scale)

No	Question	Avg	STD
1.	Co-design lab helped me deepen my knowledge in the field	4.5	0.548
2.	Due to the co-design lab, I will remember more of what I have learned	4.33	0.816
3.	Co-design lab significantly improved my understanding of the subject content	4.33	0.516
4.	Co-design lab motivated me and made me commit myself more to the subject	4.17	0.983
5.	Co-design lab helped me figure out what is the most important in the subject	4.5	0.5483
6.	As a learning experience, the co-design lab was more productive than listening to a lecture	4.67	0.516
7.	As a learning experience, the co-design lab was more enjoyable than listening to a lecture	4	1.095
8.	The co-design lab should not be assigned to future classes	1.67 **(R)**	0.816
9.	The learning experience provided by the co-design lab was not worth the effort	1.5 **(R)**	0.837
10.	The co-design lab gave me stronger motivation to work hard at learning than listening to lectures does	4	0.894

Their experience has been surveyed, and positive results have been found. The questionnaire contained a total of 26 questions that were answered using a 5-point Likert scale:

1 - Strongly Disagree,
2 - Disagree,
3 - Neutral,
4 - Agree,
5 - Strongly Agree.

An excerpt of the results that refers to students' experience with new learning approach is presented in Table 2, in which questions 8 and 9 are reversed (control questions) - R. From this survey, it can be observed that this approach helped students to better understand the subject content and get better insight into its essence. Furthermore, students evaluated this approach as more productive and enjoyable than listening to a classical lecture. Overall satisfaction of students can be qualified as very positive. They also have an affirmative attitude regarding the inclusion of the developed approaches to future classes.

6. CONCLUSION

In this paper, we presented a VR-based workflow that provides an interactive and technologically enhanced approach to teaching and learning complex assembly sequence design methods. The integration of VR technology enables students to engage directly with virtual models, allowing them to explore product structure and gain a deeper understanding of the liaisons between product parts, precedence relations, and feasible assembly sequences. Moreover, the use of this technology facilitates the teaching and learning of procedures for implementation of complex Bourjault and Cut-set methods, particularly in situations where the physical product is not available to students and abstract reasoning becomes a barrier to understanding.

The opportunity for hands-on interaction in a virtual environment, combined with the encouragement of critical and self-critical thinking, makes this workflow a valuable support tool in engineering education. It also plays an important role in achieving the intended learning outcomes. The results of the students' survey show that they are highly satisfied with the learning outcomes obtained and that they prefer this way of teaching to classical lectures.

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