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# MATHEMATICAL MODEL OF PNEUMATIC CARTESIAN ROBOT

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***Abstract:*** *This paper presents the development of a mathematical model for a robot with a Cartesian structure, commonly used in industrial and automation systems due to its simplicity, accuracy, and ease of control. The robot's mechanical design is based on three linear, orthogonal axes (X, Y, and Z), allowing straightforward positioning in three-dimensional space. The study focuses on the formulation of the kinematic and dynamic models necessary for precise control and trajectory planning. The dynamic model is derived using Newton-Euler methodology. Special attention is given to the structural configuration and the movement constraints imposed by the linear pneumatic actuators (three rodless cylinders, one cylinder with a piston rod on one side and a gripper). The resulting model provides a clear mathematical framework for understanding the behaviour of the Cartesian robot under various operating conditions. This work lays the foundation for future implementation of control algorithms and integration into automated systems, and it serves as a reference for the design and analysis of similar robotic platforms.*

***Keywords:*** *Cartesian structure, Robot, Pneumatic, Mathematical Model.*

## 1. INTRODUCTION

A pneumatic Cartesian robot is a type of robotic system that moves along three linear axes — X, Y, and Z — using pneumatic cylinders powered by compressed air, figure 1. Unlike traditional electric-driven robots, pneumatic Cartesian robots rely on air pressure to create motion, offering a fast and simple solution for many industrial tasks [1].

The construction of these robots is based on a straightforward mechanical structure: each axis is equipped with a pneumatic actuator that allows movement in a straight line. The frame, often made of aluminium or steel, provides rigidity and precision to the system. Pneumatic valves control the flow of air, enabling the robot to extend or retract actuators at high speeds. Movement is usually directed through a basic control system, sometimes integrated with sensors to improve position accuracy.

One of the main advantages of pneumatic Cartesian robots is their speed. Because air-powered actuators can respond almost instantly, these robots are ideal for high-cycle applications such as picking and placing parts, sorting small objects, or moving lightweight products on assembly lines. Their design is also relatively cost-effective, both in terms of initial investment and maintenance, making them a popular choice in industries that prioritize quick operation and simple automation [2,3].

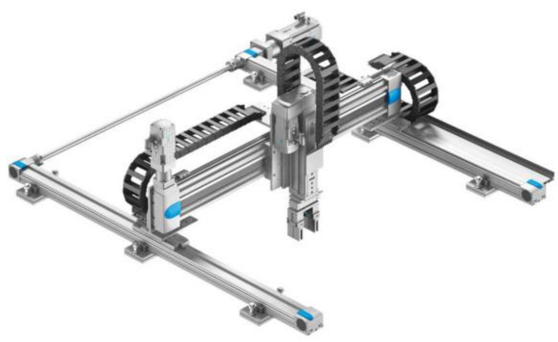

**Figure 1.** Pneumatic Cartesian robot

However, pneumatic systems have limitations, especially regarding precision and energy efficiency. Since air is compressible, fine control of motion is harder to achieve compared to electric-driven robots. This can lead to variability in positioning, which is not suitable for tasks requiring high accuracy. In order to better control the robot, it is very important to have a good mathematical model.

Despite these challenges, pneumatic Cartesian robots excel in environments where speed, reliability, and simplicity are more critical than pinpoint accuracy [3]. They are commonly used in packaging, food processing, laboratory automation, and light manufacturing. With proper design and integration, these robots can greatly enhance productivity while keeping system complexity and costs low.

## 2. ARCHITECTURE OF A PNEUMATIC CARTESIAN ROBOT

The architecture of a pneumatic Cartesian robot refers to its mechanical structure, motion system, and control components, all working together to achieve linear movement along three perpendicular axes (X, Y, and Z) using compressed air as the power source, figure 2. This type of robot is designed for straightforward, fast, and reliable handling of tasks in industrial automation.

The robot's frame is typically built from aluminium profiles or steel beams to provide rigidity and support for all moving parts. The frame is laid out in a rectangular, gantry-like structure, ensuring stability during fast linear movements.

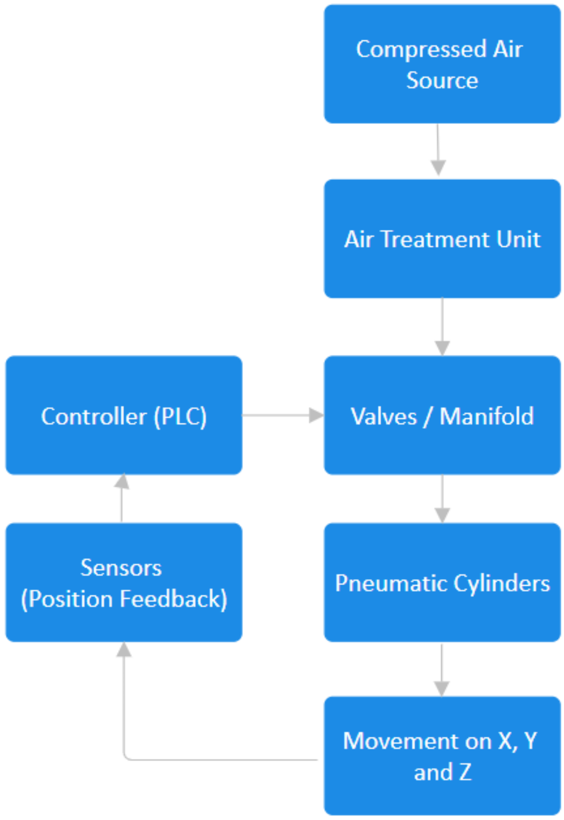


**Figure 2.** The architecture of a pneumatic Cartesian robot

Each axis is equipped with linear guides or sliding rails. These rails keep the motion aligned and reduce friction, allowing the pneumatic actuators to move the load precisely and smoothly.

The movement along each axis (X, Y, Z) is driven by pneumatic linear actuators. When compressed air is introduced into the cylinder, it pushes or pulls the piston, causing linear displacement.

The system needs a compressed air source that delivers clean, dry air at controlled pressures. Air treatment units are often used to ensure reliable actuator performance.

Proportional control valves manage airflow into and out of the cylinders. Those valves are commonly used to switch air direction based on control signals.

The robot may be controlled by a Programmable Logic Controller (PLC) or a dedicated pneumatic controller. This unit sends signals to the proportional valves to control actuator movement.

Sensors are placed along the axes to detect the position of moving parts, ensuring safe and accurate operation.

The end of the robot usually carries a simple gripper, suction cup, or another tool adapted to the specific task. Pneumatic actuators often power these tools as well.

Emergency stops, pressure monitors, and safety barriers are incorporated to protect operators and equipment.

Flexible energy chains organize and protect the pneumatic tubes and sensor wires as the robot moves along different paths.

## 4. MATHEMATIC MODEL OF PNEUMATIC CARTESIAN ROBOT

A pneumatic cartesian robot consists of three rodless cylinders, one cylinder with a piston rod on one side and a gripper on the end, figure 3.

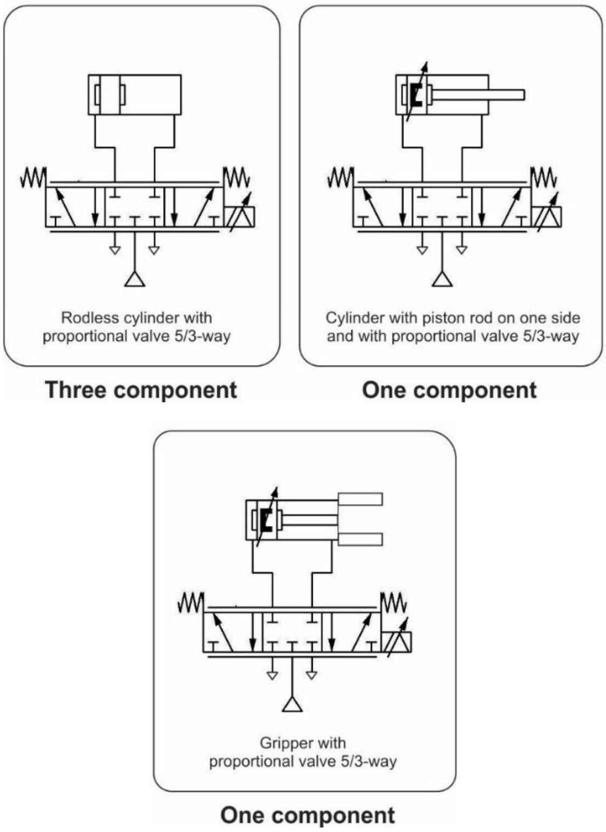


**Figure 3.** Pneumatic Cartesian robot

The dynamic equation of each rodless cylinders can be best represented by the motion equation (1):

$$(M_L + M_P)\ddot{xp} + \beta\dot{xp} + F_f + F_L == (P_1 - P_2)A \quad (1)$$

where: $M_L$ - the mass of external parts connected to the cylinder rod, $M_P$ - the piston and rod assembly mass, *xp* - the piston position in *x* or *y* or *z* direction (coordinate axis), *β* - the viscous friction coefficient, $F_f$ - the Coulomb friction force, $F_L$ - the external force, $P_1$ and $P_2$ - the absolute pressures in the cylinder's chambers, *A* - the piston effective area.

The dynamic equation of cylinder with a piston rod on one side and gripper with two fingesrs can be best represented by the motion equation (2):

$$\begin{aligned}(M_L + M_P)\ddot{xp} + \beta\dot{xp} + F_f + F_L &= \\ = P_1A_1 - P_2A_2&\end{aligned} \quad (2)$$

where: $A_1$ and $A_2$ - the piston effective areas on the left and right sides of cylinders.

Dynamics of the pressure change in thepneumatic cylinder chambers can be represented by (3):

$$\begin{aligned}W &= \frac{RT}{V_{0i} + A_i\left(\frac{1}{2}L \pm x\right)} \\ ALFA &= \alpha_{in}\dot{m}_{in} - \alpha_{out}\dot{m}_{out} \\ \dot{P}_i &= W[ALFA \pm \alpha P_i A_i \dot{x}]\end{aligned} \quad (3)$$

where: *R* - the ideal gas constant, *T* - temperatures, $\dot{m}_{in}$ and $\dot{m}_{out}$ - the mass flows entering and leaving the chamber, respectively, $\alpha$, $\alpha_{in}$, $\alpha_{out}$- the coefficients taking the values between 1 and *κ*, depending on the heat transfer process [4], *L* - the piston stroke, $i$=1, 2 the cylinder chambers index, $V_{0i}$ - the inactive cylinder volumes. In this form the pressure equation change accounts for the different heat transfer processes of the charging and discharging processes, air compression or expansion during the piston motion, the difference in effective areas on the opposite sides of the piston and the inactive volumes at the ends of stroke.

The first term in equation (3) represents the effect on pressure of the gas flow in or out of the chamber while the second term of the equation accounts for the effect of piston motion [4,5].

Each of the cylinders, as well as the gripper, is controlled by proportional 5/3 valves with electromagnetic activation.

In order to simplify the problem of designing the mathematical model of the valve, it is assumed that the movements of the valve armature and the piston are small and that it is a static problem.The displacement of the piston of the proportional valve depending on the activating current signal can be described as:

$$\boldsymbol{x_v} = K_1 u \tag{4}$$

where: $K_1$ – valve coefficient and $u$ - a current control signal from PLC. Depending on the size of the control signal, the piston of the proportional valve moves to one side or the other, thus changing the active orifice area of the valve.

Mass flow through proportional valve orifice can be represented by:

$$U = c_f A_v C_1 \frac{P_{up}}{\sqrt{T}} \tag{5}$$

$$J = \frac{P_{down}}{P_{up}} \tag{6}$$

$$\dot{m}_v = \begin{cases} UC_1, & J \leq 0{,}528 \\ UC_2 J^{1/k} \sqrt{1 - J^{\frac{k-1}{k}}}, & J > 0{,}528 \end{cases} \tag{7}$$

where: $\dot{m}_v$- the mass flow through proportional valve orifice, $\boldsymbol{c_f}$ - nondimensional coefficient, $P_{up}$ and $P_{down}$ - are upstream and downstream pressure in valve, $A_v$ – represents active orifice area of the valve and its value depends on the magnitude of the valve activation signal. The dependence of the active orifice area of the valve on the activation signal is explained in detail in the paper [5].

Constants $C_1$ and $C_2$ are depending of fluid, in our case it is compressed air ($k = 1{,}4$) and $C_1 = 0{,}040418$ and $C_2 = 0{,}156174$, [5].

## 5. CONCLUSION

This study presented the development of a mathematical model for a Cartesian-structure robot, focusing on the analytical formulation of its kinematic and dynamic characteristics. The model provides a structured framework for understanding the system's motion behavior and serves as a foundation for future analysis and optimization. Although no simulations were conducted within the scope of this paper, the derived equations and theoretical framework can be directly applied in computational environments to evaluate performance under various operational scenarios. Future work will involve validating the model through numerical simulations and experimental testing to confirm its accuracy and applicability in real-world robotic systems.

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