Society of Production Engineering

ICPES 2025

40th INTERNATIONAL CONFERENCE ON PRODUCTION ENGINEERING - SERBIA 2025

DOI: 10.46793/ICPES25.084K

University of Nis Faculty of Mechanical Engineering

Nis, Serbia, 18 - 19th September 2025

ANALYSIS OF THE IMPACT OF FRICTION ON THE QUALITY OF REGULATION DEPENDING ON THE USED FRICTION MODEL AND THE CONTROL LAW

Igor KOCIĆ[1], Saša S. NIKOLIĆ[1]*, Darko MITIĆ[1], Nikola DANKOVIĆ[1], Staniša PERIĆ[1], Miloš MADIĆ[2], Milan TRIFUNOVIĆ[2]

Orcid: 0009-0002-8569-1180; **Orcid:** 0000-0003-2745-3862; **Orcid:** 0000-0001-9483-0176; **Orcid:** 0000-0003-0986-5695; **Orcid:** 0000-0003-4766-195Xc; **Orcid:** 0000-0002-2310-2590; **Orcid:** 0000-0002-4653-6957;

[1]Univerity of Niš, Faculty of Electronic Engineering, Niš, Republic of Serbia

[2]Univerity of Niš, Faculty of Mechanical Engineering, Niš, Republic of Serbia

*Corresponding author: sasa.s.nikolic@elfak.ni.ac.rs

Abstract: *This paper analyses the effects of friction, i.e. the friction model, on control performance. The analysis was carried out using the model of the control object and the friction model with the application of different control laws. The Stribeck and LuGre friction models were considered. Position responses, control signals and friction force are shown as a function of the control laws of PID, SMC and TS LQR controllers. All simulations were performed using the MATLAB software package. The possibility of modeling a nonlinear system with friction using the Takagi-Sugeno fuzzy system and the method of generating a control signal using an LQR controller for each individual linear subsystem based on parallel distributed control (PDC) are shown. Control signals are generated using PID, SMC and TS-LQR control laws respectively. Instead of setting poles for each individual rule of the TS controller, the LQR method is used at the level of each individual rule by selecting unique Q and R matrices based on optimality criteria.*

Keywords: *friction, Striebeck model, LuGre model, PID, SMC, Takagi-Sugeno LQR, friction compensation.*

1. INTRODUCTION

In this paper, a simulation analysis of the effect of friction on the quality and performance of the regulation was performed based on the selected friction model. Knowledge of the effects of friction and its compensation is very important for design and control of servo mechanisms, robots, and precise pneumatic and hydraulic systems. Due to its nonlinearity, it causes a static error, leads to oscillation around the set value as a result of a rapid transition from the range of static to the range of dynamic friction due to the difference between the force of static and dynamic friction and the occurrence of limit cycles. The control object used is a simplified model of a DC motor with independent excitation, which is controlled by the current in the rotor and drives the positioning system via a gearbox. All simulations were performed with Matlab and Simulink using the friction models of Stribeck and LuGre. The analysis of the effect of friction on the performance of the controlled variable as well as the compensation method was carried out using the classical PID controller, which is considered the standard industrial controller, then the SMC (Sliding Mode Controller) controller and finally the Takagi-Sugeno controller (TSLQR) was used. The model of the nonlinear system is derived using the Euler-Lagrange equations. The TS model was created using fuzzy if-then rules according to the local TS approximation method, which is based on the properties of TS fuzzy models as universal approximators. Based on the obtained fuzzy model, the fuzzy controller was designed using the same if-then rules, where the control signals were obtained based on the given optimality criterion and the parallel distributed control (PDC) method based on the TS fuzzy model. Position responses, control signals and friction force are presented as a function of the friction models, control laws and compensation methods used.

2. SYSTEM MODEL

The mathematical model of the nonlinear system (1) was obtained by applying the Euler-Lagrange method and known expressions for Coulomb [1] and viscous friction (1), Stribeck (2) and the LuGre friction model (3) [2-4], where F_c is the Coulomb friction force, F_s is the maximum static friction force, F_v is the viscous friction, x is position, $\dot{x}$ velocity, v_s is Stribeck velocity, δ is a coefficient that depends on the type and scope of the model (δ=2 was taken in the paper), σ_0, σ_1 are respectively the coefficients of strength and damping of asperity fibers, σ_2 is the coefficient of viscous friction, z is a variable for the LuGre model, which refers to the amount of stretching of asperity fibers in the simulation model. In (5), the controlled system model is given with the LuGre friction model based on which the simulation was performed, a similar controlled system model was created for the Stribeck friction model.

$$F_{tr}(\dot{x},\theta) = F_c sgn(\dot{x}) + F_v$$
$$F_v = \sigma_2 \dot{x} \qquad (1)$$

$$F_{tr}(\dot{x},\theta) = \left[F_c + (F_s - F_c)e^{-\left(\frac{|\dot{x}|}{|v|_s}\right)^{\delta}}\right] sgn(\dot{x}) + \sigma_2 \dot{x}, \qquad (2)$$

$$g(\dot{x}) = F_c + (F_s - F_c)e^{-\left(\frac{|\dot{x}|}{|v|_s}\right)^{\delta}}, \delta = 2$$
$$\frac{dz}{dt} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z$$

$$F_{tr}(\dot{x},\theta) = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 \dot{x}. \qquad (3)$$

The system model is given in the general form (4), x is the position (linear or angular), F_{tr} is the friction force, which depends on the speed, $\dot{x}$ and the parameters Θ describing the friction model used, W refers to the mass or inertia, k_m is the motor amplification coefficient.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}\\ -W^{-1}F_{tr}(\dot{x},\theta)\end{bmatrix} + \begin{bmatrix}0\\ W^{-1}k_m\end{bmatrix} u. \qquad (4)$$

The simulation models are based on Fig. 1, where the controller depends on the PID simulation [5-7], the SMC controller [8, 9], with and without set point compensation, the TS LQR controller [9], and the control object is a DC motor and the friction model is Stribeck or LuGre.

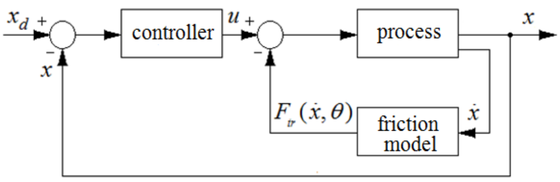

Figure 1. Block diagram of the control object and the controller

The control object and the friction model are given by equation (5), in which the variables are given in the following order: x_1=x position, x_2=v velocity, x_3=i_a rotor current, x_4=z internal variable describing the elastic deformation of the asperity fibres according to the LuGre friction model:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\sigma_1\left(x_2 - \frac{|x_2|}{g(x_2)}x_4\right) - \sigma_2 x_2 - \sigma_0 x_4 + k_m x_3 \\
\dot{x}_3 &= -ax_2 - bx_3 + cu \\
\dot{x}_4 &= x_2 - \frac{|x_2|}{g(x_2)}x_4 \\
g(x_2) &= F_c + (F_s - F_c)e^{-\left(\frac{x_2}{v_s}\right)^2}.
\end{aligned} \quad (5)$$

The analysis of the effect of friction was performed on the basis of simulations made using a system simulation model and a friction model with different control laws. First, a simulation was carried out with the PID controller, the Stribeck model and the LuGre model, without and with pre-filtering of the setpoint.

3. ANALYSIS OF THE INFLUENCE OF FRICTION FORCE

We use a classical PID controller to control the position x (either linear or angular), x_d is the setpoint of the position. The parameters of the PID controller were chosen to show the influence of friction and the model used on the quality of the control. The friction models Stribeck and LuGre were used and the controlled system model (5) with the parameters given in the Table 1 partly taken from [3]. Figures 2 and 3 show the response of the position to the step signal when using the Stribeck and LuGre models, with the classic PID (6) used as the controller, the setpoint is shown in blue and the achieved position value in red colour. Response according to Fig. 2 is similar to the response without the friction model, since the Stribeck model provides the response at low speeds. In Fig. 3 we can see that due to the transition from the static friction region to the dynamic friction region, there is a permanent oscillation around the set value (error at steady state), which leads to the occurrence of limit cycles which are not seen in Fig. 2. This phenomenon and the dynamics are described by the LuGre model, in contrast to the Stribeck model, which does not describe them.

Table 1. Parameters of controlled system

Parameter	Value
σ_0	10^4 N/m
σ_1	100 Ns/m
σ_2	4 Ns/m
F_c	10 N
F_s	15 N
v_s	0.001m/s
Δ	2
Ra	4.58 Ω
La	0.0281 H
Ke	1.25 Vs/rad
Km	1.25 Nm/A

$$u = k_p\left(e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{de(t)}{dt}\right) \quad (6)$$
$$e = x_1 - x_d$$

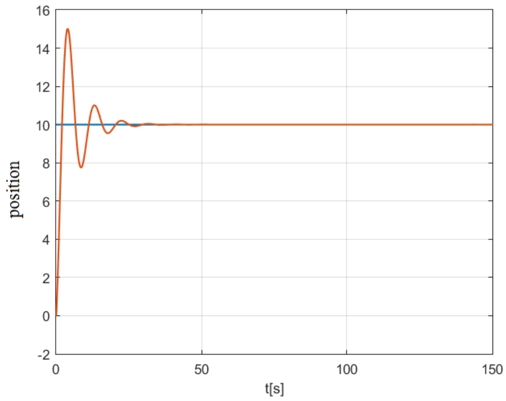

Figure 2. Response of the position when using the Stribeck model and the classic PID controller

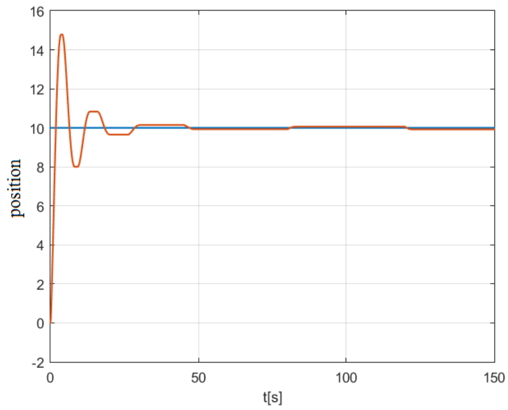

Figure 3. Response of the position when using the LuGre model and the classic PID controller

The use of a compensator (pre-filter) of the set value or simply the use of a ramp of the set value reduces the jump in the position response, it can even be completely eliminated, but the effect of the limit cycles increases.

Figures 4 and 5 show the friction force and the control signal when using a classic PID controller without compensation of the position setpoint and the LuGre friction model, where the oscillation of both the friction force and the control signal is clearly visible. The control signal must oscillate in order to maintain the position response.

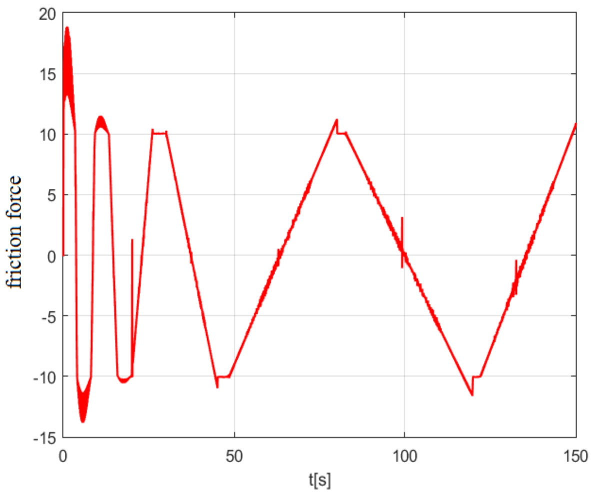

Figure 4. Friction force when using the Lugre model and the classic PID controller

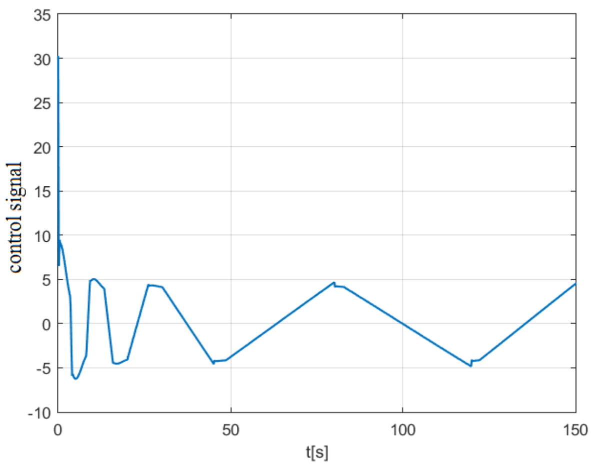

Figure 5. Control signal when using the Lugre model and the classic PID controller

In the further analysis, we used the SMC controller with exponential reaching law (7), Stribeck and LuGre friction model. The sliding surface *s* is given by (8), where *e* is the deviation error of the current value $x=x_1$ (the difference of the position from the system model, x_1 (5) and the set value of the position, x_d (9)). By replacing the corresponding equations from (5) and arranging the expressions, the control law of the SMC controller is obtained in the form of (10), where the expressions for T_f and X_d are given as (11) and (12) respectively.

$$\dot{s} = -k_d s - k\, sgn(s(x)), \tag{7}$$

$$s = \left(\frac{d}{dt} + \lambda\right)^{n-1} e, \tag{8}$$

$$e = x_1 - x_d, \tag{9}$$

$$u = \frac{1}{c}(bx_3 + 2\lambda T_f + (a-\lambda)x_2 + \\ + X_d - k_d s - k\,\mathrm{sgn}(s)), \tag{10}$$

$$T_f = \sigma_1(x_2 - \frac{|x_2|}{g(x_2)} x_4) + \sigma_2 x_2 + \sigma_0 x_4, \tag{11}$$

$$X_d = \ddot{x}_d + 2\lambda\ddot{x}_d + \lambda\dot{x}_d. \tag{12}$$

A very fast response, similar to an aperiodic response without jumps, is observed for both applied models, although the response is faster when the LuGre model is applied. Chattering is observed in the control signals in both cases, which could be reduced by using the function *tg*(*s*) instead of *sgn*(*s*). Figures 6 and 9 show that the friction forces are fully compensated after a few seconds, they do not have a pronounced oscillatory character as when with a PID controller, the setpoint is shown in blue and the achieved position value in red colour. Figure 7 shows the friction force when using the Stribeck model. It can be seen that, in contrast to the PID control law, the friction force does not have a pronounced occurrence of oscillations. Figure 8 shows the control signal when the Stribeck model is applied, where chattering is noticeable.

The control error is almost completely absent, the steady-state error is reduced to almost zero and the limit cycles are eliminated. All this is reflected in the use of robust nonlinear controllers such as the SMC. An additional setpoint compensator is not required. Figure 10 shows the friction force when the LuGre model is applied. In contrast to the PID control law, figure shows that the friction force has no oscillations. Figure 11 shows the control signal when using the LuGre model, where chattering is also noticeable.

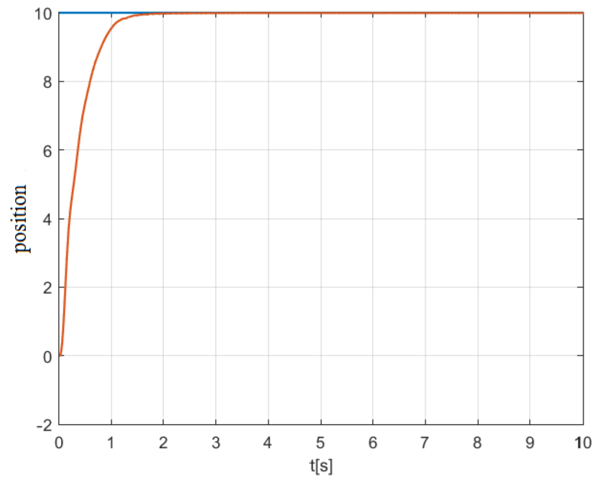

Figure 6. Response of the position when using the Stribeck model and the SMC controller

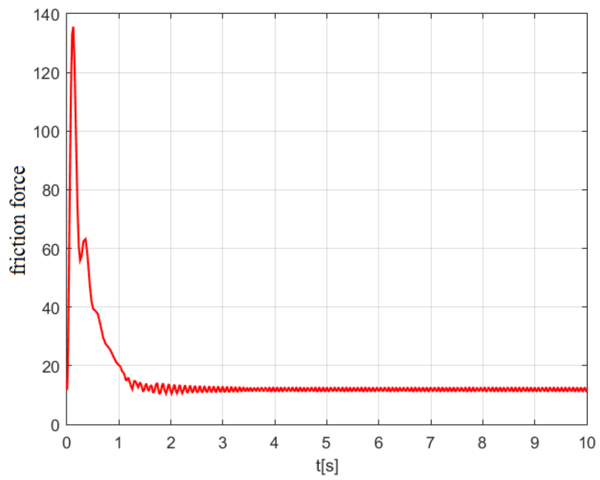

Figure 7. Friction force when using the Stribeck model and the SMC controller

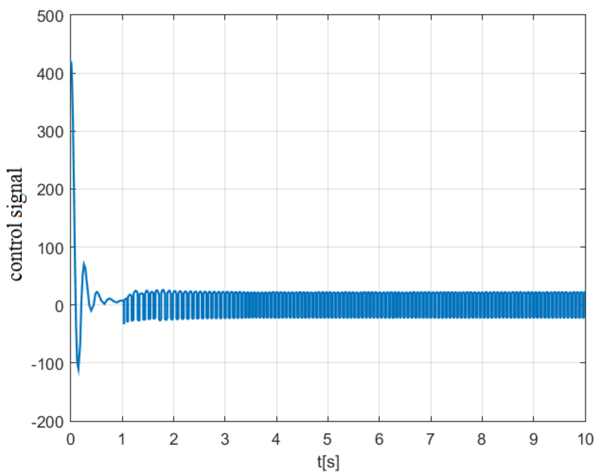

Figure 8. Control signal when using the Stribeck model and the SMC controller

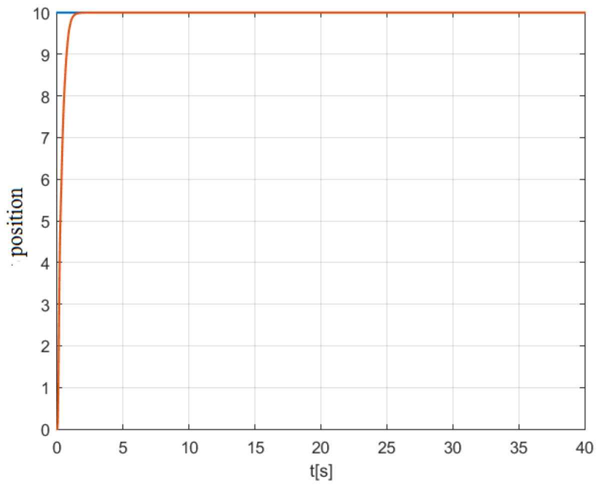

Figure 9. Response of the position when using the LuGre model and the SMC controller

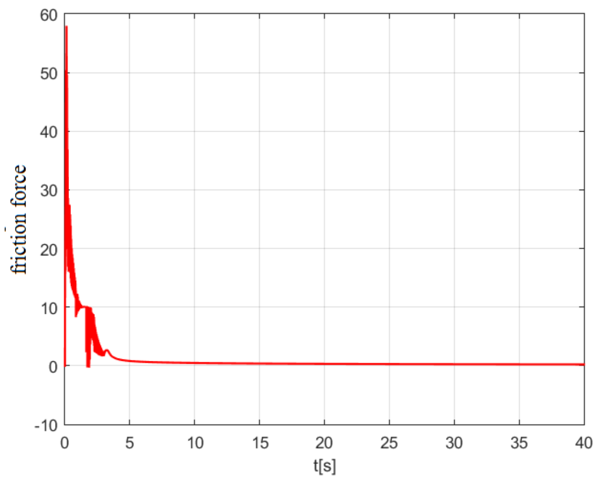

Figure 10. Friction force when using the Lugre model and the SMC controller

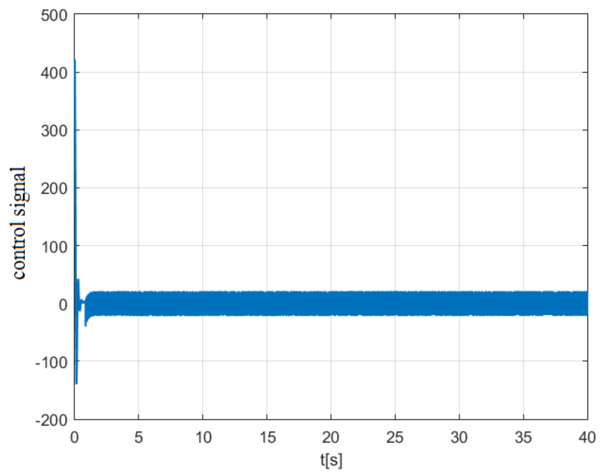

Figure 11. Control signal when using the LuGre model and the SMC controller

4. TAKAGI-SUGENO LQR CONTROL

The analysis of the influence of the friction force and its compensation was performed by modelling the nonlinear system (5) using the TS fuzzy system, which is expressed in the form of several local linear dynamic systems of the form (13) with m rules (R_1-R_m). An approach based on local approximation in fuzzy space was used by replacing the nonlinear terms in (5) with suitably chosen linear terms, which in the case of system 1 (5) reduces the rules to the state variable x_2, since the nonlinearities accordingly appear in the form of an exponential function and an absolute value [3]. The membership functions $\mu_{Pij}(x_i)$ of the fuzzy sets $P_{10}, ..., P_{nm}$ are suitably selected functions of the triangular or Gaussian type.

The activation degree and the fuzzy basis function $j=1,...,m$ of this rule is given by the expression (14), where the nonlinear system (5) is approximated by the TS fuzzy system (15) and the control law is generated using the parallel distributed control (PDC), replacing the linear matrix inequality (LMI) method to determine the stability of the system thus constructed by the LQR method.

$$R_j: IF\, x_1 \text{ is in the area of } P_{1j} \text{ and } \dots \text{ and } x_n \text{ is in the area of } P_{nj}$$

$$THEN\ \dot{x}_i = A_i x + B_i u \tag{13}$$

$$\mu_j(x) = \prod_{i=1}^{n} \mu_{Pij}(x_i)$$
$$\xi_j = \frac{\mu_j(x)}{\sum_{j=1}^{m} \mu_j(x)} \tag{14}$$

$$\dot{x}(t) = \frac{\sum_{j=1}^{m} \mu_j(x(t))(A_j x(t) + B_j u(t))}{\sum_{j=1}^{m} \mu_j(x(t))} =$$
$$= \sum_{j=1}^{m} \xi_j(x(t))\,(A_j x(t) + B_j u(t)). \tag{15}$$

For each individual rule of the TS fuzzy controller, control is designed on the basis of a specific performance index based on the Q and R matrices. For this purpose, the *lqr* command is used in MATLAB, where the Q and R matrices are selected by trial and error. The control law was generated by determining the gain matrix k_j for each individual rule (13), which was obtained from the linear quadratic optimization problem (16) and for each individual linear system based on the fuzzy model (5).

$$J = \int_{t_0}^{t_f} (x^T Q x + u^T R u) dt, \tag{16}$$

$$u = -\left[\sum_{j=1}^{m} \xi_j\,(t) k_j\right] x(t). \tag{17}$$

The total control signal is obtained as the sum of the product of the fuzzy basis function and the local control signals obtained within the if-then rules of the fuzzy controller of the obtained PDC control (17). Based on the obtained control laws $u_j(t)$, the TS controller is designed. Using the local TS approximation, the model (5) for velocities around zero reduces to (18), which is exactly one of the rules for the TS model for low velocities, i.e. velocities close to zero.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(\sigma_1 + \sigma_2) x_2 + k_m x_3 - \sigma_0 x_4 \\ \dot{x}_3 &= -a x_2 - b x_3 + c u \\ \dot{x}_4 &= x_2. \end{aligned} \tag{18}$$

A similar law was found in [10, 11]. The parts of the rules for velocities around Stribeck are derived in a similar way.

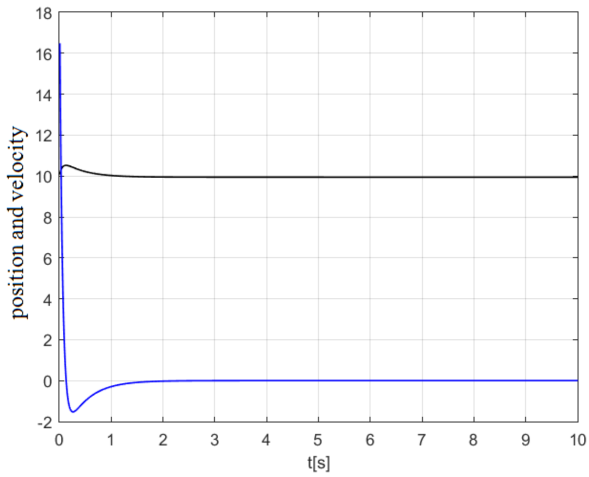

Figure 12. Control signal when using the LuGre model and the TS LQR controller

The simulation was done using the 4*th* order Runge-Kuta method, for which a routine was written in MATLAB.

Figure 12 shows the position response and the speed of the position change when using the TS LQR controller. It can be observed that the friction forces are fully compensated after one second, which is a consequence of using the TS LQR controller and approximating the nonlinear friction force model by a linear one.

5. CONCLUSION

The simulations carried out on the basis of Stribeck and LuGre friction model show that the LuGre model describes the friction of mechanical systems much better than Stribeck model. Therefore, the LuGre model is regularly used in industrial friction compensation methods and in the identification of friction parameters. All newer thyristor controllers have an integrated friction compensation option. Based on the simulation analysis during position control when using the PID controller, a step in the position response as well as an oscillating behaviour of the friction force and the control variables is observed. The position pre-filter reduces the jump, but increases the oscillatory force of the friction force and the control variables. In the simulation with the SMC controller, good friction compensation was achieved and the error in the steady state of the position was eliminated, and chattering in the control signal was observed as the only drawback.

Significantly better position response in terms of speed and overshoot was obtained in the simulation with the TSLQR controller, where the linearization of the nonlinear system was performed with the fuzzy model. The proposed controller was obtained by determining the gain matrix for each individual fuzzy rule using the LQR theory. The proposed controller was obtained by determining the gain matrix for each fuzzy rule using LQR theory. It remains to be analysed in future works on a real mechanical system with a TSLQR controller and a LuGre type fuzzy friction model, without and with a friction force compensation based on this model.

ACKNOWLEDGEMENT

This work was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia [grant number 451-03-137/2025-03/200102 and 451-03-137/2025-03/200109].

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