ICPES 2025

40th INTERNATIONAL CONFERENCE ON PRODUCTION ENGINEERING - SERBIA 2025

Society of Production Engineering

DOI: 10.46793/ICPES25.036K

University of Nis Faculty of Mechanical Engineering

Nis, Serbia, 18 - 19th September 2025

CNC TURNING DYNAMICS: MODELLING WITH NEWTON'S LINEAR INTERPOLATION AND MATLAB

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Abstract: *This paper presents a mathematical model for linear interpolation motion in CNC turning, developed empirically using Newton's linear interpolation formula. The model provides a precise representation of CNC lathe motion, offering practical insights into the dynamic behaviour of CNC turning operations. By implementing this model in MATLAB, a graphical solution was generated, which simultaneously enables visualisation and facilitates a deeper understanding of CNC machining processes and their optimisation. The study emphasises the significance of mathematical modelling in CNC machining, demonstrating how Newton's linear interpolation formula can effectively predict and represent tool movement along a workpiece. The empirical approach of the model ensures the accuracy and reliability of CNC lathe simulations, contributing to improved precision and efficiency in turning operations. The MATLAB implementation further enhances the model's applicability by enabling detailed graphical analysis, making it a useful tool for both theoretical research and practical applications in automated machining.*

Keywords: *Turning operation, CNC lathe, CNC interpolator, Interpolating polynomial, Interpolated value.*

1. INTRODUCTION

CNC turning is a sophisticated machining technique designed to shape metals by removing the excess material from a rotating workpiece. The process begins with securing the starting workpiece, typically a cylindrical rod or blank, in CNC lathes. The CNC lathes rotate the workpiece around its central axis, a critical feature that allows the cutting tool to interact effectively with the material, enabling precise material removal and machining [1].

The cutting tool is controlled by a Computer Numerical Control (CNC) system that operates on a series of instructions written in G-code. The G-code instructions direct the cutting tool's movements and the rotational speed of the workpiece, allowing the CNC system to control the cutting tool along defined paths and contours. A crucial component of a CNC lathe is the CNC interpolator, which converts G-code instructions into accurate tool paths and motion commands, ensuring smooth and precise movements. This precise control is essential for achieving exact material removal, ensuring uniformity, and ultimately obtaining the desired shape and dimensions of the workpiece [2].

CNC turning is one of the most frequently used metal-cutting operations, determined by several fundamental principles. During this operation, the workpiece rotates (Fig. 1a) while the cutting tool applies shear force to remove the excess material, forming thin layers known as chips (Fig. 1b) [3].

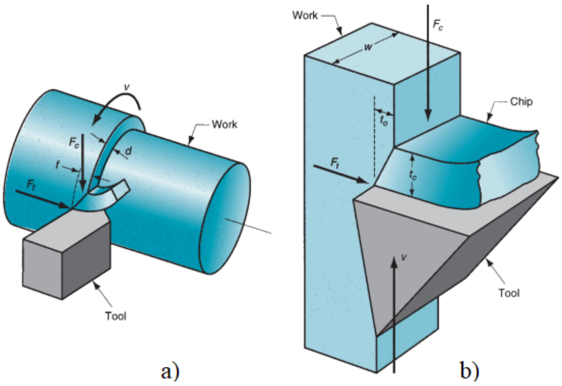

Figure 1. a) Turning operation, b) Chip formation [4]

The efficiency of the cutting action depends on factors such as the hardness of the metal, the sharpness of the cutting tool, the cutting speed, and the feed. The cutting speed refers to the distance, measured in metres, that the circumference of the rotating workpiece travels as it passes the cutting edge of the cutting tool in one minute. The feed, on the other hand, is the distance that the cutting tool advances in an axial direction during each rotation of the workpiece, also measured per minute. Both cutting speed and feed rate are crucial for determining the toolpath and attaining the desired contour and surface finish of the final product [5].

Mathematical modelling, or modelling, holds significant importance in CNC turning for generating and controlling toolpaths. This process uses mathematical equations to define the trajectory of the cutting tool. One of the modelling concepts in CNC turning is linear interpolation, which involves moving the cutting tool in a straight line between two points in the workspace (Fig. 2a). This method is essential for creating precise straight cuts and features, requiring the calculation of intermediate points along the cutting toolpaths and adjustments to their positions. Linear interpolation ensures smooth and accurate cutting tool movement, maintaining the desired cutting parameters [6].

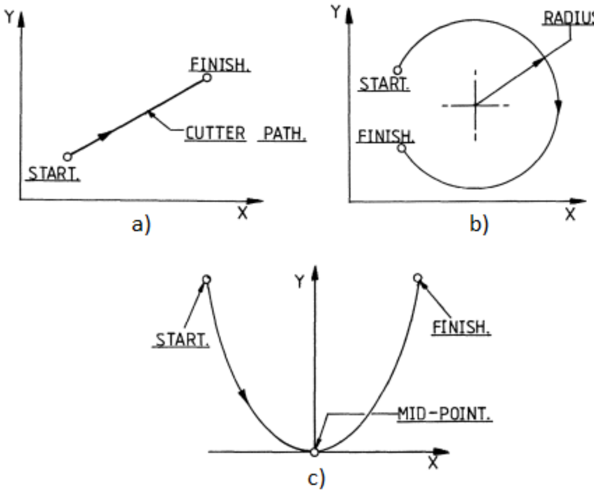

Figure 2. a) Linear interpolation, b) Circular interpolation, c) Parabolic interpolation [7]

When turning more complex profiles, CNC systems utilise circular interpolation (Fig. 2b) and parabolic interpolation (Fig. 2c) to machine arcs or circular profiles. This technique involves mathematical methods to determine the toolpath's curvature, which is crucial for producing final parts with round features or curved surfaces [8].

The solution and visualisation of the mathematical modelling in CNC turning operations are greatly facilitated by MATLAB. It provides robust tools for graphical solutions and simulations, enabling engineers and machinists to model and visualise toolpaths and material removal processes effectively. With MATLAB, users can input the mathematical equations that define the toolpaths, and the software generates graphical representations of these paths. This visual approach aids in optimising the toolpath design, predicting potential issues, and applying adjustments to improve the machining process. Additionally, MATLAB's capabilities allow for the analysis of various scenarios and parameters before actual machining begins. This leads to better development and implementation, ultimately improving the efficiency and accuracy of the CNC turning operations [9].

In this context, CNC turning is a highly precise and efficient machining technique that integrates advanced mathematical modelling and interpolation methods to control toolpaths on modern CNC lathes. By utilising linear interpolation methods for mathematical modelling and employing MATLAB for graphical solutions, the accurate and consistent interpolation of intricate shapes by CNC lathe interpolators can be effectively illustrated. These terms are explained and discussed in relevant literature (e.g., [1, 9]). The paper is structured into four sections: an introduction, an explanation of the research, a demonstration and discussion of the results, and a conclusion with potential applications.

2. RESEARCH

In this research, the linear interpolated motion within the context of machining symmetrical workpieces, with a certain emphasis on contour precision and design specifications, is analysed. A detailed example, illustrated in Figure 3, serves as the focal point of this analysis.

The particular workpiece intended for turning operation on a CNC lathe is designed as a chess piece (Fig. 3a). This model was chosen for its inherent symmetry along the longitudinal axis, which exemplifies the complexity and considerations of turning symmetrical objects.

Figure 3c complements this by detailing the dimensions and proportions of the chess piece, offering a clear and detailed view of its design specifications. These specifications are critical for understanding the interpolation required in the CNC turning operation.

To determine the exact path that the cutting tool will follow, Figure 3b presents a set of coordinates labelled from A to K. These coordinates draw the precise toolpath characterised by linear interpolation. This means the cutting tool moves in straight lines from one coordinate to the next, ensuring a smooth transition from point A to point K.

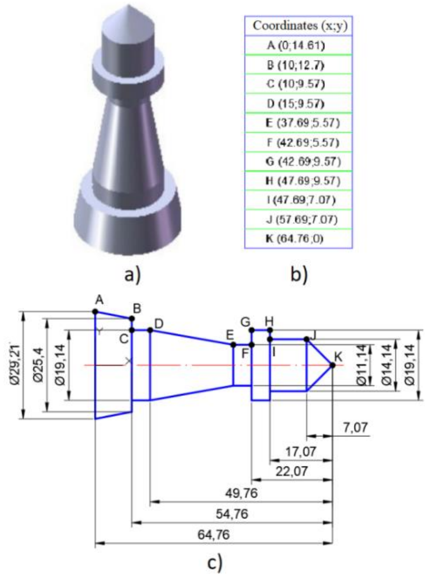

Figure 3. a) 3D model of the workpiece [10], b) Coordinates, c) Dimensions

The linear interpolated motion involves a series of straight-line movements between defined points. At each point, the cutting tool changes direction as it travels along a straight segment connecting consecutive points. This method ensures precision and control in forming the toolpath, which is crucial for achieving the desired geometry and symmetry of the workpiece.

The analysis in this research employs Newton's linear interpolation formula. This formula estimates function values at points within a discrete set of known values. It is a special case of polynomial interpolation where the interpolating polynomial is of degree one (linear).

Given two points with coordinates (x_i;y_i) and (x_{i+1};y_{i+1}), the linear interpolating polynomial P(x) that passes through these points is:

$$P_i(x) = y_i + (x - x_i) \cdot \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \quad (1)$$

Here, P(x) represents the estimated value of the function at the point x; x_i and x_{i+1} are the x-coordinates of the given points; and y_i and y_{i+1} are the corresponding y-coordinates.

The Newton's linear interpolation formula assumes that x lies within the interval [x_i;x_{i+1}], which defines the range over which the function is approximated. This interval specifies that the interpolation will only be accurate for x values within [x_i;x_{i+1}]. Outside this range, the linear interpolation may not provide reliable estimates.

3. RESULTS AND DISCUSSION

This section presents the results from the analytical calculation for the linear interpolated movement between the defined points using Newton's formula for linear interpolation. The process involved several key steps, including the identification of the points, the calculation of intervals, and the application of the interpolation formula. The results are systematically organised and presented in Table 1, which includes data for the interval number, the starting and ending points of each interval, the defined intervals, and the interpolated polynomial values at specified points within each interval.

Table 1. Results from the linear interpolation analysis

INT. No.	Parameter	Value
1	Starting Point	A(0;14.61)
	Ending Point	B(10;12.7)
	Interval	[0;10]
	Polynomial P(x)	**$P_0(x) = 14.61 - 19.1x$**
2	Starting Point	B(10;12.7)
	Ending Point	C(10;9.57)
	Interval	[10;10]
	Polynomial P(x)	**/**
3	Starting Point	C(10;9.57)
	Ending Point	D(15;9.57)
	Interval	[10;15]
	Polynomial P(x)	**$P_2(x) = 9.57$**
4	Starting Point	D(15;9.57)
	Ending Point	E(37.69;5.57)
	Interval	[15;37.69]
	Polynomial P(x)	**$P_3(x) = 12.27 - 0.18x$**
5	Starting Point	E(37.69;5.57)
	Ending Point	F(42.69;5.57)
	Interval	[37.69;42.69]
	Polynomial P(x)	**$P_4(x) = 5.57$**
6	Starting Point	F(42.69;5.57)
	Ending Point	G(42.69;9.57)
	Interval	[42.69;42.69]
	Polynomial P(x)	**/**
7	Starting Point	G(42.69;9.57)
	Ending Point	H(47.69;9.57)
	Interval	[42.69;47.69]
	Polynomial P(x)	**$P_6(x) = 9.57$**
8	Starting Point	H(47.69;9.57)
	Ending Point	I(47.69;7.07)
	Interval	[47.69;47.69]
	Polynomial P(x)	**/**
9	Starting Point	I(47.69;7.07)
	Ending Point	J(57.69;7.07)
	Interval	[47.69;57.69]
	Polynomial P(x)	**$P_8(x) = 7.07$**
10	Starting Point	J(57.69;7.07)
	Ending Point	K(64.76;0)
	Interval	[57.69;64.76]
	Polynomial P(x)	**$P_9(x) = 64.76 - x$**

First, the specific coordinates of the points (x_i,y_i) and (x_{i+1},y_{i+1}) used for the interpolation were identified. (These points represent the known values between which interpolation will occur.) Next, the intervals between consecutive points were calculated.

The Newton's linear interpolation formula was then applied to estimate the values between the given points. Using equation (1), the interpolating polynomial P(x) at any given x within the interval $[x_i,x_{i+1}]$ was calculated and is provided in Table 1. When the x-values in the interval were identical, the interpolation step was avoided, and the y-value remained constant, representing a simple movement along the axis without any need for calculation. Additionally, the coordinates of the defined points were input into a MATLAB script implementing Newton's linear interpolation formula. The script processes the x and y coordinates, applies interpolation to estimate intermediate values, and generates a graphical representation. The corresponding MATLAB script is presented in Figure 4 below.

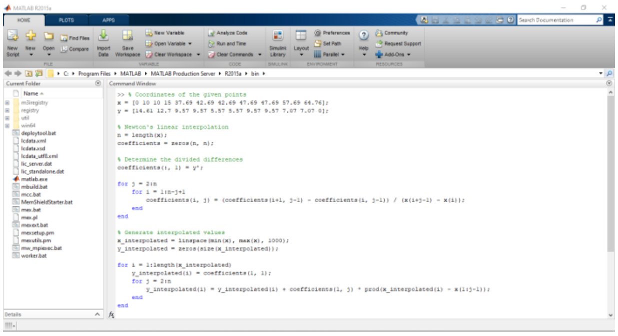

Figure 4. Overview of the MATLAB script

Following the successful implementation of the interpolation model, the MATLAB script generates an accurate graphical representation of the results. This visualisation, shown in Figure 5, effectively illustrates the linear interpolation applied to machine the workpiece through the turning operation, corresponding to the coordinates defined in Figure 3b.

The graphical solution offers a detailed view of the linear interpolation method functioning within the CNC turning operation. This method ensures precise adjustments and refinements during the operation, which is crucial for maintaining the desired accuracy and quality in the final machined product. By utilising the specific coordinates from Figure 3b, the interpolation model translates these points into a smooth and continuous trajectory that represents the path of the cutting tool for CNC turning.

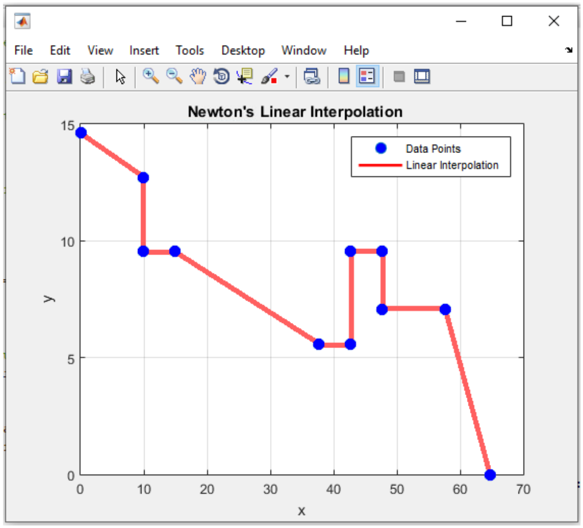

Figure 5. The graphical solution overview

This visual representation accomplishes several critical purposes. Firstly, it confirms the accurate application of the interpolation model by displaying a seamless transition between the points that aligns perfectly with the desired turning path. This verification ensures that the model performs as expected and adheres to the planned trajectory.

Secondly, the graphical depiction is a valuable asset for further analysis, offering engineers and technicians a detailed view of the effectiveness of the interpolation model in real-world scenarios. By examining the manner in which the linear interpolation aligns with the intended CNC turning strategy, they can identify areas for improvement and make necessary adjustments to enhance performance.

Furthermore, the graphical representation plays a key role in optimising the CNC turning operations. It clearly illustrates the relationship between coordinates and the interpolation toolpath, facilitating the identification of any discrepancies or potential areas for improvement. This clarity can lead to greater precision in machining operations, reduced material waste, and overall improved efficiency. The visualisation also enhances communication by translating complex data into a more accessible format. Mechanical engineers, including project managers, machinists, and operators, can better understand the complexities of CNC turning operations and the role of interpolation in achieving high-quality final parts. This representation thus serves as a bridge between theoretical models and practical applications, fostering a deeper understanding and collaboration among all involved mechanical engineers and departments.

The results obtained validate both the outer contour of the designed model and the toolpath generated by MATLAB. This validation clarifies the function of the CNC interpolator, demonstrating how the developed mathematical model supports the simulation results in CNC machine tools. It reveals the intricate relationship between the CNC interpolator and the CNC simulator, offering insights into their operational mechanisms in CNC lathes.

4. CONCLUSION

In this paper, a mathematical model for linear interpolation motion in CNC turning was analysed using Newton's linear interpolation formula. The formulation and implementation of the model in MATLAB provided significant insights into the dynamic behaviour of CNC lathes during CNC turning operations, offering a valuable tool for both theoretical understanding and practical application in CNC machining.

The application of Newton's linear interpolation formula in the development of this model demonstrated an effective method for representing and predicting the motion of CNC lathes. The MATLAB implementation of the model additionally enhanced its practical utility by facilitating graphical solutions and visualisations, allowing for a deeper understanding of the model's behaviour and its implications for real-world CNC turning.

Therefore, this empirical method has achieved a clear and precise representation of the linear interpolation motion, which is crucial for enhancing the accuracy and efficiency of CNC turning operations.

Moreover, the importance of mathematical modelling in CNC machining was underscored by this research, particularly in the understanding and optimisation of the dynamic performance of CNC lathes. The insights gained from this model can inform future developments in CNC technology, improve the precision of turning operations, and contribute to advancements in automated machining processes. Additionally, the integration of this model with MATLAB provided a powerful platform for ongoing research and development in this field, enabling further exploration and improvement of the model.

In conclusion, the development of this linear interpolation model provides a robust foundation for both theoretical exploration and practical application. Future research may build upon this model to explore more complex interpolation techniques, integrate additional factors affecting CNC turning, and further expand the mathematical framework to enhance the capabilities of CNC machining processes.

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